Multi-sensor activation for temporally correlated event monitoring with renewable energy sources

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Abstract: Future sensor networks would comprise sensing devices with energy-harvesting capabilities from renewable energy sources, such as solar power. This paper focuses on design of efficient algorithms for multi-sensor activation to optimise overall event detection probability in presence of uncertainties in event and recharge processes. We formulate the dynamic multi-sensor activation question in a stochastic optimisation framework, and show that a time-invariant threshold policy, which maintains an appropriately chosen number of sensors active at all times, is optimal in absence of temporal correlations. Moreover, the same energy-balancing time-invariant threshold policy approaches optimality in presence of temporal correlations as well, albeit under certain limiting assumptions. We also analyse the class of correlation-dependent threshold policies and derive the range for energy-balancing thresholds. Through simulations, we compare the proposed time-invariant policy with energy-balancing correlation-dependent policies, and observe that although the latter may perform better, the performance difference is rather small in the cases studied.

Keywords: multi-sensor activation; temporal correlations; energy constraints; energy-harvesting sensor systems.


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1 Introduction

Wireless sensor networks are deployed for detecting interesting phenomena in a wide range of environments, including oceans, forests, atmosphere and military-surveilled regions. Typically, individual sensors are heavily constrained in terms of resources such as computational power and energy. The tiny, low-cost nature of the sensing devices along with their minimal processing capabilities creates the need to develop simple but efficient algorithms for their operations. In addition, the energy usage at the sensors must be optimised to improve performance of the sensor network.

For long-term monitoring of the targeted environments, sensors are envisioned to be deployed with rechargeable batteries, which are capable of harnessing energy from renewable sources in the environment. For instance, Heliomote (Hsu et al., 2005; Raghunathan et al., 2005), a solar energy-harvesting platform, demonstrates the self-sustaining capability of a sensing device. Moreover, the proliferation of non-rechargeable-battery-based sensors causes severe environmental hazards and advocates the need for green-technology-based solutions. Sensing devices capable of harvesting solar power (Hsu et al., 2005; Jiang et al., 2005; Raghunathan et al., 2005; Norman, 2007) and other energy sources, including wind and vibration energy (MicroStrain, 2003), utilise the high availability of a renewable energy source to enable near-perpetual operation of the sensor network. One of the most important issues in the efficient operation of such sensor networks lies in the design of intelligent store-and-use energy-harvesting frameworks for energy management (Kansal and Srivastava, 2003; Kansal et al., 2004). Design of energy-efficient algorithms for sensor operations is vital towards realisation of such frameworks.

Sensors are often, by design, unreliable and energy inefficient. This is because it is quite costly to build highly reliable and energy-efficient sensors. For instance, a sensor may need to be equipped with a huge solar panel for it to have high energy availability at all times. In addition, optimal deployment of sensors is not feasible in many application scenarios such as environmental monitoring and battlefield surveillance. Typically, sensors are small, energy-constrained devices with low recharge rates (dependent upon energy harvesting) and may have to spend a significant fraction of their lifetime in inactive or ‘sleep’ state. Moreover, the individual sensors are prone to failures. Therefore, in practice, due to cost efficiency and feasibility of deployment, sensors need to deployed randomly and redundantly (at a high density) in the region of interest to guarantee reliability in the sensing and communication processes.

For better performance, these sensors would need to work collaboratively to achieve a global network objective, such as reliable event detection and reporting. In the generalised event detection application that we consider in this paper, application-specific events, which the system is required to detect, occur randomly in the region of interest and can potentially exhibit temporal correlations across their occurrences. The overall system objective is to maximise the time-average event detection probability in the system. The discharge of an active sensor depends on the activation algorithm as well as on the event occurrence process, while the recharge is based upon harnessing renewable energy. We address the following multi-sensor activation question in a renewable energy based sensor system – How should the sensor nodes be activated so as to optimise the overall event detection probability achieved by the system?

Sensor nodes would typically operate under uncertain operational conditions, including unknown energy replenishment schedules, partial system state information and varying degrees of spatio-temporal correlations in the sensed phenomena. These factors add a new dimension to the design of efficient algorithms for sensor sleep scheduling (Kar et al., 2006; Jaggi et al., 2008, 2009), transmission (Borkar et al., 2005; Zhang and Chanson 2005), routing (Lin et al., 2005), active energy management (Niyato, Hossain, Rashid et al., 2007; Vigorito et al., 2007) and rate allocation (Fan et al., 2008) questions. In this paper, we use the sensor energy model first proposed in Jaggi et al. (2009) (for single-sensor activation question) to formulate and solve the multi-sensor activation question under temporally correlated event phenomena. The overall objective of the designed algorithms is to guarantee high availability of the network in the presence of uncertainties in sensed phenomena and in energy replenishment schedules.

The main contributions of this paper include:

- Formulation of the multi-sensor activation question, while appropriately modelling the uncertainties involved in the event occurrences and in the renewable energy sources.
- Designing activation schedules for multi-sensor systems to maximise the overall event detection probability in the presence of temporal correlations.
- Analysing two different classes of threshold activation policies to evaluate their performance under various system parameters.
- Proposing a time-invariant threshold policy and demonstrating its near-optimal performance and its robustness to the presence of uncertainties in the system.

The paper is organised as follows. Next we discuss the modelling of uncertainties and formulate the problem as a dynamic optimisation question in Section 2. Section 3 discusses various activation algorithms considered and primarily focuses on threshold-based policies. Section 4 analyses the performance of proposed algorithms and presents simulation results. Section 5 discusses related research in renewable-energy-based sensor systems among others. Finally, we summarise the conclusions and future directions in Section 6.

2 Problem formulation

In this section, we elaborate upon the sensor energy model used to characterise the operations of a renewable-energy-based sensing device. We then discuss the event occurrence process used to model the event phenomena. We also present the application-specific performance metric, which is later used in the performance evaluation of the designed algorithms.
2.1 Sensor energy model

The energy bucket of a rechargeable sensor stores energy in units of a quantum. The size of the sensor energy bucket is denoted by $K$. The presence of uncertainties in renewable-energy-based recharge process is modelled using a stochastic framework. We assume a discrete time model where in each time slot, a recharge event occurs with a probability $q$ and charges the sensor with a constant charge amount of $c$ quanta. The recharge processes at the different sensor nodes are assumed to be independent of each other; however, the parameters $c$ and $q$ are the same across all sensors.

The discharge rate at the sensor depends upon its activation (inactivation) state, as well as the state of the application-specific event phenomena. A sensor having non-zero energy level is said to be in active (inactive) state if it has been activated (deactivated) in the current time slot. A sensor having zero energy level is said to be in the dead state. The sensor expends a charge amount of $\delta_i$ quanta (operational cost) during each time slot it is active. In addition, if an application-specific event is detected by the active sensor, the sensor expends an additional charge amount of $\delta_2$ quanta (detection and transmission cost). We assume that $\delta_i \geq c$ and $\delta_2 \geq \delta_i$. We also assume that a sensor can be activated only if it has sufficient energy to operate successfully for at least one time slot, i.e., its energy level is at least $\delta_i + \delta_2$. Note that we assume that a sensor discharges energy only when in active state; however, the analysis can be extended to consider non-zero energy discharge in inactive state.

The discharge rate of a sensor (per time slot) equals $qc$. Similarly, the discharge rate of an active sensor is given by $\delta_i + \delta_2 \pi p_d$, where $\pi$ denotes the steady-state probability of occurrence of an application-specific event during the times the sensor is active, and $p_d$ denotes the event detection probability of the sensor. Typically, the recharge rate of a sensor would be significantly less than its discharge rate in the active state, which necessitates the design of an efficient activation algorithm for sensor operations.

2.2 Application-specific event phenomena

We model the application-specific event phenomena which the sensor system is required to detect and report as a correlated stochastic process in order to characterise the inherent randomness and temporally correlated event occurrence. For example, consider a sensor network deployed to detect and warn against forest fires. If the temperature in any region in the forest rises above 100°F, it might represent the possibility of a forest fire. Now, if the temperature at some point of time is higher than this threshold, then with high probability, it would remain above this threshold in near future as well. Similarly, if the temperature is much below alarming levels (the above threshold), then it is likely to remain so in the immediate future. Thus, smart activation algorithms should take into consideration the state (and correlation information) of the application-specific event phenomena while deciding upon activation schedules.

The extent of temporal correlation in application-specific event phenomena is specified using correlation probabilities $p_{on}$ and $p_{off}$ such that $\frac{1}{2} \leq p_{on}, p_{off} \leq 1$. If an event occurs during time slot $t$, then in the next time slot $(t+1)$, a similar event occurs with probability $p_{on}$, while no such event occurs with probability $1 - p_{on}$. Similarly, if no event occurred during the current time slot, no such event occurs in the next time slot with probability $p_{off}$. The event occurrence process used to model the application-specific event phenomena comprises an alternating sequence of periods where events occur (On period) and do not occur (Off period). In practice, application-specific events would occur rarely; therefore, the Off periods are expected to be significantly larger than the On periods, which implies $p_{off} \geq p_{on}$. Nevertheless, our analysis applies to scenarios where $p_{off} < p_{on}$ as well. Note that since $p_d$ is a measure of reliability of the sensor (and hence is a property of the sensor node), whereas the correlation probabilities $p_{on}$ and $p_{off}$ are measures of temporal correlations in the event phenomena (and hence are a property of the application-specific event phenomena), we assume that $p_d$ is independent of $p_{on}$ and $p_{off}$. Figure 1 depicts the sensor discharge/recharge model and behaviour of an individual sensor during different states of the event process.

Consider a time slot $t$ such that an event occurred during time slot $t-1$ but no event occurred during time slot $t$. Let $X$ denote the random variable representing the number of time slots (including $t$) after which the event occurs again. Then, $Pr[X = i] = (p_{on})^{i-1}(1 - p_{off}), \forall i \geq 1$. Therefore:

$$
E[X] = \frac{1 - p_{off}}{(1 - p_{off})^2} = \frac{1}{1 - p_{off}}.
$$

Thus, the expected length of an Off period in the event occurrence process is given by $\frac{1}{1 - p_{off}}$. Similarly, the expected length of an On period equals $\frac{1}{1 - p_{on}}$. Using Markov chain analysis, the steady-state probability of event occurrence equals $\pi_{on} = \frac{1 - p_{off}}{2 - p_{on} - p_{off}} (\pi_{off} = 1 - \pi_{on})$. 

Multi-sensor activation for temporally correlated event monitoring
Consider a system of \( N \) identical renewable-energy-based sensing devices deployed in a region of interest to monitor an application-specific event phenomena. If an application-specific event occurs during a time slot, each sensor independently detects the event with a probability \( p_d \) (event detection probability). Note that the detection probability of a sensor depends upon its distance from the target of event occurrence (Chin and Hu, 2008). Typical values of individual detection probabilities are expected to lie in the range [0–0.5] (Chin and Hu, 2008). Since we consider sensor deployments and event occurrences to be random, it is reasonable to assume that the detection probabilities of different sensors are independent of each other. Also, since each time, the event is expected to occur at a different location in the region of interest, the average event detection probability of an individual sensor is modelled as \( p_d \).

Although the sensors may be located at different points in space, since the event occurrences are random, we assume that the events are equally likely to occur anywhere (in a uniformly distributed fashion) in the region of interest. Similar arguments could be used for the energy-harvesting (recharge) process as well which justify the assumption that the sensors are identical in their sensing capabilities and in their recharge/discharge dynamics, over a large period of time.

Now, if \( n \) out of the \( N \) sensors were in active state during the above time slot and an event occurred during the time slot, let the overall event detection probability achieved be denoted \( U(n) \). In general, \( U(n) = 0 \) when \( n = 0 \), and increases with \( n \). Figure 2 depicts the performance of the system during an arbitrary time slot.

Note: Since there are three active sensors, the event detection probability in the system during time slot \( t \) equals \( U(3) \). Since there are four sensors with positive energy, the maximum achievable detection probability during time slot \( t \) equals \( U(4) \). Maximum detection probability during any time slot equals \( U(5) \).
Two examples of feasible utility functions are provided below:

Example 1: Let \( n \) sensors be active and an event occurs during time slot \( t \). Then, the probability that the event gets detected by at least one active sensor is given by

\[
\hat{U}(n) = 1 - (1 - p_d)^n.
\]

Note that the overall event detection probability \( \hat{U}(n) \) is zero when no sensor is active during the time slot, and increases as the number of active sensors \( n \) increases from 0 to \( N \). In other words, the utility function \( \hat{U}(n) \) is a non-decreasing and strictly concave function, with \( \hat{U}(0) = 0 \).

Figure 3 depicts the shape of this utility function for various values of sensor event detection probability \( p_d \).

Example 2: Let \( n \) out of the \( N \) sensors be active during time slot \( t \). If an event occurs during time slot \( t \), each active sensor independently detects the event with probability \( p_d \).

Let \( n_d \) denote the number of sensors which (correctly) detect the event during time slot \( t \). The sensor system declares event detected if \( n_d > \frac{n}{2} \), i.e., more than half of the active sensors detect the event. Thus, event detection probability during time slot \( t \) equals \( \Pr[n_d > \frac{n}{2}] \). We have:

\[
\Pr[n_d = i] = \binom{n}{i} (p_d)^i (1 - p_d)^{n-i} \forall i \in [0 \ldots n].
\]

\[
\Pr\left[ n_d > \frac{n}{2} \right] = \sum_{i=\lceil \frac{n}{2} \rceil}^{n} \binom{n}{i} (p_d)^i (1 - p_d)^{n-i}.
\]

Using Chernoff’s bound (Motwani and Raghavan, 1995), when \( p_d > 0.5 \), this majority-decision rule function is lower bounded by \( \hat{U}(n) \) as:

\[
\Pr\left[ n_d > \frac{n}{2} \right] \geq \hat{U}(n) = 1 - e^{-2\delta(p_d - \frac{1}{2})^2}.
\]

Note that \( \lim_{n \to \infty} \hat{U}(n) = 1 \). Figure 4 plots this utility function \( \hat{U}(n) \) as a function of number of active sensors. We observe that this utility function \( \hat{U}(n) \) is also a non-decreasing and concave function.

Note that we do not explicitly assume the definition of the utility function \( U(n) \) in our analysis, and our results apply to all applications where the performance can be expressed using a non-decreasing and concave utility function \( U(n) \), including the above examples.

Figure 4 Generalised utility function representing the majority-decision rule (see online version for colours)

The goal of the system is to maximise its event detection capability over time. Let \( \Pi \) denote the activation algorithm (or policy) employed by the sensor system. Let \( n_i^\Pi \) denote the number of active sensors in the region during time slot \( t \) when the sensor system operates under policy \( \Pi \). Let \( \vec{n}^\Pi \) denote the vector \([n_1^\Pi, n_2^\Pi, \ldots] \). Let \( x_i \) be the indicator variable denoting the occurrence of an event during time slot \( t \), i.e., \( x_i = 1 \) if an event occurred during time slot \( t \); 0 otherwise. Then, the performance of policy \( \Pi \), denoted \( \bar{U}(\Pi) \), is given by:

\[
\bar{U}(\Pi) = \lim_{T \to \infty} \sum_{t=1}^{T} x_t U\left(n_t^\Pi\right) / \sum_{t=1}^{T} x_t.
\]

The decision problem is that of finding the activation policy \( \hat{\Pi} \) such that \( \hat{\Pi} = \arg \max \bar{U}(\Pi) \).

3 Activation algorithms/policies

A threshold policy with parameter \( m \) is characterised as follows. An available sensor (i.e. a sensor with energy level \( \geq \delta_1 + \delta_2 \)) is scheduled for activation in a time slot if the number of sensors scheduled for activation during this time slot is less than the threshold \( m \); otherwise, the sensor is moved to inactive state until the next decision instant.
A new decision is taken if the threshold parameter changes, if any active sensor runs out of energy and moves to the dead state or if a sensor that was previously in the dead state becomes available through battery recharge. Thus, a threshold policy with a threshold of \( m \) tries to maintain the number of active sensors in the system as close to \( m \) as possible (however never exceeding \( m \)).

In view of the temporally correlated nature of the application-specific event phenomena, smart threshold policies might employ two different threshold parameters: possibly a larger threshold during the On periods, and a smaller threshold during the Off periods. We consider two different threshold policies, namely time-invariant threshold policy (TTP) and correlation-dependent threshold policy (CTP). The TTP algorithm is oblivious to the temporal correlation information and employs a constant threshold parameter at all times. On the other hand, the CTP algorithm employs different threshold parameters during the On and Off periods, respectively:

- **Time-invariant threshold policy (TTP):** During each time slot, a threshold of \( m \) active sensors is targeted from a set of available sensors. A TTP algorithm is simpler to use in practice. Since the threshold parameter does not vary over time, it requires minimal state maintenance overhead at the sensing devices.

- **Correlation-dependent threshold policy (CTP):** A threshold of \( m \) is employed in time slot \( t \) if the event occurrence process is known to be in the On period, i.e., if an application-specific event was detected in the previous time slot \( t-1 \) by any of the active sensors. Otherwise, a threshold of \( n \) \((\leq m) \) is targeted in time slot \( t \). Thus, CTP algorithm applies a time-varying threshold as opposed to a constant threshold employed by the TTP algorithm. Intuitively, the CTP algorithm tries to conserve the energy at the sensors during the Off periods in order to be able to use it more judiciously during the On periods. Note that the CTP algorithm is a special case of the CTP algorithm with \( n = m \).

Note that all the above activation algorithms are simpler to deploy in a sensor network since they require minimal state information and can be realised based only upon local information. Of particular interest are the activation algorithms which achieve energy balance in the renewable-energy-based multi-sensor system in steady state. For instance, a TTP algorithm with parameter \( m_{TTP} \) achieves energy balance if the average recharge rate in the system equals the average discharge rate in the sensor system when the threshold parameter \( m_{TTP} \) is applied. Similarly, multiple CTP threshold pairs \((m, n)\) could achieve energy balance in the sensor system. The algorithms which achieve energy balance in the sensor system achieve better performance, as we show in the next section. Intuitively, this is similar to maintaining the service rate in a queueing system to be equal to the arrival rate, thus achieving the maximum possible utilisation.

### 4 Performance analysis

The performance achieved by an activation policy \( \Pi \) is measured using equation (5). In this section, we derive an upper bound on achievable performance in Section 4.1. We then analyse the performance of various threshold policies in Section 4.2 and present simulation results in Section 4.3.

#### 4.1 Upper bound on optimal performance

Since the optimal performance is difficult to characterise, we obtain an upper bound on it. We later compare the performance of our proposed activation algorithm with respect to this bound.

Let \( \psi_{i,t} \) be the indicator variable denoting whether the sensor \( i \) was active during time slot \( t \), i.e., \( \psi_{i,t} = 1 \) if the sensor \( i \) was active during time slot \( t \); 0 otherwise.

**Lemma 1:** For all sensors \( i \in \{1...N\} \):

\[
\lim_{T \to \infty} \sum_{t=1}^{T} x_{i,t} \psi_{i,t} \leq \left( \frac{1 - \Pi}{\delta_i + \delta_c p_c^m} \right) \left( p_c^m \right) \]

**Proof:** As \( T \to \infty \), the total number of application-specific events that occur during time \([1...T]\) satisfies

\[
\lim_{T \to \infty} \sum_{t=1}^{T} x_{i,t} = T \Pi \]

Let \( P_{i,t} = \Pr[x_i = 1 | \psi_{i,t} = 1] \). Then:

\[
P_{i,t} = \frac{1}{\Pr[\psi_{i,t} = 1]} \left[ \Pr[x_i = 1, \psi_{i,t} = 1 | x_{i-1} = 1] \Pr[x_{i-1} = 1] + \Pr[x_i = 1, \psi_{i,t} = 1 | x_{i-1} = 0] \Pr[x_{i-1} = 0] \right] = \frac{1}{\Pr[\psi_{i,t} = 1]} \left[ p_c^m \Pr[\psi_{i,t} = 1 | x_{i-1} = 1] + (1 - p_c^m) \Pr[\psi_{i,t} = 1 | x_{i-1} = 0] \right]
\]

\[
= \frac{1}{\Pr[\psi_{i,t} = 1]} \left[ \Pr[x_{i-1} = 1 | \psi_{i,t} = 1] + (1 - p_c^m) \Pr[\psi_{i,t} = 1 | x_{i-1} = 0] \right] \leq \frac{1}{\Pr[\psi_{i,t} = 1]} \left[ p_c^m + p_c^m (1 - p_c^m) \right] \]

\[
= \frac{p_c^m}{\Pi} \leq \frac{1}{1 - p_c^m}
\]

The inequality above follows since \( p_c^m + p_c^m (1 - p_c^m) \geq 1 \), and hence \( 1 - p_c^m \leq \frac{1}{p_c^m} \). Also, since the event occurrence process is independent of sensor activation states, we have used the following equality above: \( \Pr[x_i = 1 | \psi_{i,t} = 1, x_{i-1} = 1] = \Pr[x_i = 1] \)

Let \( T_i \) denote the number of time slots in which the sensor \( i \) was active, operating under some stationary policy \( \Pi \) during time \([0...T]\). Let \( L_i \) denote the energy level of sensor \( i \) at time \( t \). The expected energy level of sensor \( i \) at...
time $T$ (assuming that the sensor did not lose any charge due to its energy bucket being full when a charge quantum arrived) is given by $E[L_{t,T}] = L_{t,0} + T \psi_{\delta_{1} + \delta_{2}P_{m}P_{d}}$, where $P_{m}$ is the steady-state probability $P_{i,t}$, i.e.:

$$P_{i,t} = \lim_{T \to \infty} \sum_{i=1}^{T} P_{i,t} \psi_{\delta_{i}}.$$  \hspace{1cm} (7)

Note that $P_{i,t} \leq p_{m}^{o} \forall t \Rightarrow P_{i,t} \leq p_{m}^{\infty}$. Since the sensor energy level is always non-negative, we have $E[L_{t,T}] \geq 0$. Simplifying, dividing by $T$, and taking the limit as $T \to \infty$, we have:

$$\lim_{T \to \infty} \frac{T}{T} \leq \frac{qc}{\delta_{i} + \delta_{2}P_{m}P_{d}}.$$  \hspace{1cm} (8)

Note that if any charge was lost due to the sensor energy bucket being full when a charge quantum arrived) is given by $\psi = \psi(\delta_{1} + \delta_{2}P_{m}P_{d})$, where $\psi$ is a non-decreasing function of $\psi_{\delta_{i}}$. From equation (6), we have:

$$\lim_{T \to \infty} \sum_{i=1}^{T} \psi_{\delta_{i}} = \lim_{T \to \infty} \sum_{i=1}^{T} \Pr[x_{i} = 1] \psi_{\delta_{i}} = \lim_{T \to \infty} \sum_{i=1}^{T} P_{i,t} \psi_{\delta_{i}} = \pi P_{i,t}.$$  \hspace{1cm} (9)

Therefore, using equation (8), we have:

$$\lim_{T \to \infty} \frac{T}{T} \leq \frac{qc}{\delta_{i} + \delta_{2}P_{m}P_{d}}.$$  \hspace{1cm} (10)

Let $U_{g}$ denote the R.H.S. in equation (10). Then:

$$\frac{dU_{g}}{dp_{m}} = \frac{1}{\pi^{m}} \frac{qc}{\delta_{i} + \delta_{2}P_{m}P_{d}} > 0.$$  \hspace{1cm} (11)

Thus, $U_{g}$ is a non-decreasing function of $P_{m}$. From equation (6),

$$U_{g} \leq \frac{1}{\pi^{m}} \left( \frac{p_{m}^{o}qc}{\delta_{i} + \delta_{2}P_{m}P_{d}} \right).$$

Now, the lemma follows:

**Lemma 2:** The performance achieved by any stationary activation policy $\Pi$ is upper bounded as:

$$\bar{U}(\Pi) \leq U \left( \frac{1}{\pi^{m}} \left( \frac{Np_{m}^{o}qc}{\delta_{i} + \delta_{2}p_{m}^{o}P_{d}} \right) \right).$$  \hspace{1cm} (11)

**Proof:** Let $f$ and $p$ be measurable functions finite a.e. on a set $\mathbb{R}$. Suppose $f \cdot p$ and $p$ are integrable on $\mathbb{R}$, $p \geq 0$ and $\int_{\mathbb{R}} p > 0$. If $\phi$ is convex in an interval containing the range of $f$, then Jensen’s inequality (Wheeden and Zygmund, 1977) states:

$$\phi \left( \int_{\mathbb{R}} f \cdot p \right) \leq \int_{\mathbb{R}} \phi(f) p.$$  \hspace{1cm} (12)

Recall that $n_{t}$ denotes the number of sensors in the active state during time slot $t$. Since $U(\cdot)$ is concave, substituting $\phi = U(\cdot)$, $f = n_{t}$ and $p = x_{i}$ in the above equation, Jensen’s inequality in the discrete-state space implies:

$$U \left( \sum_{i=1}^{T} n_{i} x_{i} \right) \leq \sum_{i=1}^{T} U(n_{i}) x_{i}.$$  \hspace{1cm} (13)

Since $U(\cdot)$ is continuous, we have:

$$\bar{U}(\Pi) = \lim_{T \to \infty} \frac{\sum_{i=1}^{T} U(n_{i}) x_{i}}{\sum_{i=1}^{T} x_{i}} \leq \lim_{T \to \infty} U \left( \sum_{i=1}^{T} n_{i} x_{i} \right) = U \left( \lim_{T \to \infty} \sum_{i=1}^{T} n_{i} x_{i} \right) = U \left( \sum_{i=1}^{T} n_{i} x_{i} \right) = U \left( \sum_{i=1}^{T} \sum_{i=1}^{T} \psi_{\delta_{i}} \psi_{\delta_{i}} \right) = U \left( \frac{1}{\pi^{m}} \left( \frac{Np_{m}^{o}qc}{\delta_{i} + \delta_{2}p_{m}^{o}P_{d}} \right) \right).$$  \hspace{1cm} (14)

The last equality follows from the fact that all sensors are identical. The last inequality follows from Lemma 1 and since $U(\cdot)$ is non-decreasing.

4.2 Performance of threshold policies

We first analyse the performance of the TTP algorithm in Section 4.2.1 and show that it achieves near-optimal performance for an appropriately chosen threshold. The energy-balancing threshold parameter $m$ is such that when this threshold is employed, the average recharge rate equals the average discharge rate in the sensor system. For simplicity of analysis, we assume infinite sensor energy bucket size (i.e. $K \to \infty$). We also assume that when $K \to \infty$, and an energy-balancing threshold parameter $m$ is employed in the system, the threshold of $m$ can be always met (i.e. with probability 1). Later, we justify this assumption by showing that for finite values of $K$, the probability that an energy-balancing threshold is not met is of the order of $O\left( \frac{1}{K^{2}} \right)$ (in Appendix A). Thus, for sufficiently large sensor energy bucket size $K$, the performance achieved
by the energy-balancing TTP algorithm is accurately characterised by the results. Note that since the granularity of discharge, in general, is of the order of milli joules (or less) (Jurdak et al., 2010), and the sensor’s battery capacity is of the order of kilo joules (or more), it is reasonable to assume that \( K \) is sufficiently large. We analyse the performance of the CTP algorithm in Section 4.2.2.

### 4.2.1 Time-invariant threshold policy

Consider a TTP employing a threshold of \( m \), denoted \( \Pi_{\text{TTP}}^{m} \). Note that the TTP algorithm with parameter \( m \) is the same as a CTP algorithm with threshold pair \((m,m)\). Consider an \( On \) period of length \( T_1 \) followed by an \( Off \) period of length \( T_2 \) in the event occurrence process. From equation (1), we have:

\[
E[T_1] = \frac{1}{1-p_c^{m_0}}, \quad E[T_2] = \frac{1}{1-p_c^{m_{\text{off}}}}.
\]

The expected amount of energy gained by the sensors in the system through recharge during this \( On/Off \) cycle, denoted \( E_i \), is given by:

\[
E_i = \frac{Nqc}{(1-p_c^{m_0})p_c^{m_0}}.
\]

(12)

Assuming that the threshold of \( m \) is always met, the expected amount of energy spent by the sensors in the system during this \( On/Off \) cycle, denoted \( E_z \), is given by:

\[
E_z = \frac{m \delta_i E[T_1] + mp_d \delta_i E[T_2]}{(1-p_c^{m_0})p_c^{m_0}}
\]

(13)

The energy-balancing TTP algorithm, denoted \( \Pi_{\text{TTP}}^{EB} \), employs a threshold such that the average recharge rate equals the average discharge rate in the sensor system. Since an \( On/Off \) cycle forms a renewal interval for the event occurrence process, the energy-balancing threshold, denoted \( m_{\text{TTP}}^{EB} \), employed by \( \Pi_{\text{TTP}}^{EB} \), can be derived using the equation \( E_z = E_i \). Thus, we have:

\[
m_{\text{TTP}}^{EB} \left[ \frac{\delta_i}{(1-p_c^{m_0})p_c^{m_0}} + \frac{p_d \delta_i}{(1-p_c^{m_0})p_c^{m_0}} \right] = \frac{Nqc}{(1-p_c^{m_0})p_c^{m_0}}
\]

\[
\Rightarrow m_{\text{TTP}}^{EB} = \frac{Nqc}{\delta_i + \delta_i p_c^{m_0} p_d}
\]

(14)

Assuming that the applied threshold is always met, the performance achieved by the activation algorithm \( \Pi_{\text{TTP}}^{EB} \) is given by:

\[
\bar{U}(\Pi_{\text{TTP}}^{EB}) = \lim_{T \to \infty} \frac{\sum_{i=1}^{T} x_i U(m_{\text{TTP}}^{EB})}{\sum_{i=1}^{T} x_i} = U(m_{\text{TTP}}^{EB}).
\]

(15)

Let \( U^* \) denote the upper bound to maximum achievable performance for any activation algorithm, given by Lemma 2. Also, let \( \beta = \frac{\delta_i}{\delta_i} \). Then, the energy-balancing TTP algorithm \( \Pi_{\text{TTP}}^{EB} \) achieves the following performance bound:

\[
\bar{U}(\Pi_{\text{TTP}}^{EB}) = U\left( \frac{Nqc}{\delta_i + \delta_i p_c^{m_0} p_d} \right) \geq \frac{\beta + \frac{1}{p_c^{m_0} p_d}}{p_c^{m_0} \delta_i} U^*.
\]

(16)

**Proof:** Since \( U() \) is a concave non-decreasing function, we have \( U(X) \geq \frac{U(Y)}{Y} \) for \( X \leq Y \). Substituting, we get:

\[
\bar{U}(\Pi_{\text{TTP}}^{EB}) = \frac{X}{Y} U(\frac{X}{Y}) \geq \frac{\beta + \frac{1}{p_c^{m_0} p_d}}{p_c^{m_0} \delta_i} U^*.
\]

(16)

Note that \( X \leq Y \) since \( p_c^{m_0} \geq \pi^m \) (because \( p_c^{m_0} + p_c^{m_{\text{off}}} \geq 1 \)). The values of \( X \) and \( Y \) above could differ substantially when the correlation probabilities are large. Note that in the absence of temporal correlations, when \( p_c^{m_0} = p_c^{m_{\text{off}}} = \frac{1}{2} = \pi^m \), from equation (16), \( \Pi_{\text{TTP}}^{EB} \) achieves optimal performance (equal to \( U^* \)).

**Corollary 1:** \( \Pi_{\text{TTP}}^{EB} \) achieves optimal performance in the absence of temporal correlations.

From Lemma 3, \( \Pi_{\text{TTP}}^{EB} \) achieves optimal performance as \( \beta \to \infty \). In practice, since transmission cost (\( \delta_i \)) is typically much larger than the sensing cost (\( \delta_i \)), \( \beta = \frac{\delta_i}{\delta_i} \) is expected to be sufficiently large. Thus, \( \Pi_{\text{TTP}}^{EB} \) achieves close to optimal performance. In fact, from Lemma 3, it achieves performance \( \geq \left( 1 - \frac{1}{\beta \pi^m p_d} \right) U^* \). We believe that the upper bound given by Lemma 2 is tight when \( p_c^{m_0} = p_c^{m_{\text{off}}} = 0.5 \), and is loose otherwise. Nevertheless, the bound is sufficiently close as seen from results in Section 4.3.
4.2.2 Correlation-dependent threshold policy

Consider a CTP employing a threshold pair of \((m, n)\), denoted \(\Pi_{CTP}^{m,n}\). Note that the CTP algorithm with parameter \(m = n\) is the same as the TTP algorithm with a threshold of \(m\). We only consider the energy-balancing threshold pair \((m,n)\) such that when they are employed, the employed threshold is always met (i.e. with probability 1). First, we derive bounds on the performance of energy-balancing CTP policies and then comment on the range of energy-balancing threshold pairs \((m,n)\).

Consider an On period of length \(T_1\) followed by an Off period of length \(T_2\) in the event occurrence process. Recall that the CTP algorithm employs a threshold of \(m\) in time slot \(t\) if the event occurrence process is known to be in the On period, i.e., if an application-specific event was detected in the previous time slot \(t-1\) by any of the active sensors. Otherwise, a threshold of \(n\) is targeted in time slot \(t\). Therefore, a threshold of \(n\) is employed during the first time slot of the On period. Similarly, a threshold of \(n\) is employed during the last \(T_2-1\) time slot of the Off period, as depicted in Figure 5. During the other time slots, a threshold of \(m\) or \(n\) is employed depending upon whether an event was detected or not during the previous time slot.

**Figure 5** Thresholds employed by CTP algorithm during a renewal interval

![Diagram showing On and Off periods with thresholds](image)

Note: Since no event was detected during the Off period, a threshold of \(n\) is employed during the first time slot of the On period. Thereafter, during the On period, a threshold of \(m\) is employed if the event was detected during the previous time slot. Otherwise, a threshold of \(n\) is employed. Similarly, during the first time slot of the Off period, a threshold of \(m\) or \(n\) is employed depending upon whether (or not) an event was detected during the previous time slot (the last time slot of the On period). Thereafter, a threshold of \(n\) gets employed during the Off period, since no event occurs (or is detected) during the Off period.

Let \(s = U(m)\), \(y = U(n)\) and \(z = s - y\). Let \(u_i\) denote the overall event detection probability achieved during time slot \(t\) of the On period. Then, \(u_0 = y\), and:

\[
u_2 = yU(m) + (1-y)U(n) = yz + (1-y)y = y + yz = y(z+1).
\]

Similarly:

\[
u_3 = u_2U(m) + (1-u_2)U(n) = yz(1+z) + [1 - y(1+z)]y = y(1+z + z^2)
\]

and so on. Thus:

\[
u_i = y(1+z + z^2 + \ldots + z^{i-1}), \forall i \in [1\ldots T_i].
\] (17)

The time average event detection probability during the On/Off cycle is given by:

\[
\sum_{i=1}^{T_i} \frac{u_i}{T_i} = \frac{y}{T_i} \left[ 1 + (1+z) + (1+z+z^2) + \ldots + (1+z+z^2 + \ldots + z^{T_i-1}) \right] = \frac{y}{T_i} \left[ 1 - z - \frac{z^{T_i}}{1-z} \right].
\]

Let \(p = \frac{1}{1-z}\). From equation (1), \(E[T_1] = \frac{1}{1-p^{on}}\). Since an On/Off cycle forms a renewal interval for the event occurrence process, the time average event detection probability achieved in the system is given by:

\[
\bar{U}(\Pi_{CTP}^{m,n}) = \frac{yz\left(1-p_n^m\right)\left(1-z^{-1/p_n^m}\right)}{(1-z)} \geq \frac{yz\left(1-p_n^m\right)}{(1-z)}.
\] (19)

Note that for the energy-balancing pair \((m,n)\), \(m = n\), \(z = U(m) - U(n) = 0\) and \(\bar{U}(\Pi_{CTP}^{m,n}) = U(n)\). In other words, \(\Pi_{CTP}^{m,n}\) reduces to \(\Pi_{TTP}^{EB}\) when \(m = n\).

Next, we consider the choices and ranges of the energy-balancing threshold pair \((m,n)\). The expected amount of energy gained by the sensors in the system through recharge during the On/Off cycle, denoted \(E_1\), is given by equation (12). Let \(v_1\) denote the expected amount of energy spent by the sensors during time slot \(t\) of the On period. Then, \(v_1 = n(\delta_i + p_j\delta_j)\), and:

\[
v_2 = [u_m + n(1-u_n)](\delta_i + p_j\delta_j).
\]

Similarly:

\[
v_i = [u_{i-1} + n(1-u_{i-1})](\delta_i + p_j\delta_j), \forall i \in [2\ldots T_i].
\] (20)

Let \(w_i\) denote the expected amount of energy spent by the sensors during time slot \(t\) of the Off period. Then, \(w_1 = [u_{t-1} + n(1-u_{t-1})]\delta_i\), and \(w_1 = w_2 = \ldots = w_2 = n\delta_i\).

Assuming that the employed threshold is always met, the total energy spent by the sensors in the system during the On/Off cycle, denoted \(E_1\), is given by:

\[
\text{Total energy spent} = \sum_{i=1}^{T_i} [u_i + n(1-u_n)](\delta_i + p_j\delta_j).
\]
Using equation (1), the expected amount of energy spent by the sensors in the system during the On/Off cycle under policy \( \Pi_{CTP}^{m,n} \), denoted \( E_z \), is given by:

\[
E_z = E[E_z] = \frac{n \delta_t}{1 - p_e} + \frac{n}{1 - p_e^m} \left( \delta_t + p_e \delta_z \right) + (m - n)p_e \delta_z y \frac{1 - z^y}{1 - z},
\]

\[
+ (m - n) \left( \delta_t + p_e \delta_z \right) \frac{y z^y}{1 - z^y} - \frac{1}{1 - z^y}.
\]

Using equation (12) and \( E_1 = E_2 \), we get:

\[
E_z = E[E_z] = \frac{n \delta_t}{1 - p_e} + \frac{n}{1 - p_e^m} \left( \delta_t + p_e \delta_z \right) + (m - n)p_e \delta_z y \frac{1 - z^y}{1 - z},
\]

\[
+ (m - n) \left( \delta_t + p_e \delta_z \right) \frac{y z^y}{1 - z^y} - \frac{1}{1 - z^y}.
\]

Using equation (1), the expected amount of energy spent by the sensors in the system during the On/Off cycle under policy \( \Pi_{CTP}^{m,n} \), denoted \( E_z \), is given by:

\[
E_z = E[E_z] = \frac{n \delta_t}{1 - p_e} + \frac{n}{1 - p_e^m} \left( \delta_t + p_e \delta_z \right) + (m - n)p_e \delta_z y \frac{1 - z^y}{1 - z},
\]

\[
+ (m - n) \left( \delta_t + p_e \delta_z \right) \frac{y z^y}{1 - z^y} - \frac{1}{1 - z^y}.
\]

The last inequality follows since \( 0 \leq z < 1 \). The best energy-balancing CTP algorithm maximises the performance objective:

\[
\max_{m,n} \sum_{\Pi_{CTP}} E_z
\]

subject to constraints:

\[
E_i = E_z,
\]

and

\[
m \geq n.
\]

The above is a non-convex optimisation problem which is not simple to solve. Hence, we focus on finding a range for the threshold parameters \( m \) and \( n \), using energy balance. Using equation (12) and \( E_1 = E_2 \), we get:

\[
\frac{N q c}{(1 - p_e^m)^{\pi_m^*}} \geq \frac{n \delta_t}{1 - p_e} + \frac{n}{1 - p_e^m} \left( \delta_t + p_e \delta_z \right) + \frac{(m - n)y}{1 - z} \left[ \delta_t + p_e \delta_z \frac{y z^y}{1 - z^y} - \frac{y z^y}{1 - z^y} \right].
\]

To simplify the expression, let us introduce the following constants. Let \( c_1 = \frac{N q c}{(1 - p_e^m)^{\pi_m^*}} \), \( c_2 = \frac{\delta_t}{1 - p_e} \) and \( c_3 = \frac{\delta_t + p_e \delta_z}{1 - p_e} \). Then, we have:

\[
c_1 \geq n(c_2 + c_3) + \frac{(m - n)y}{1 - z} \left[ \delta_t + p_e \delta_z \frac{y z^y}{1 - z^y} - \frac{y z^y}{1 - z^y} \right].
\]

Simplifying, we get:

\[
f(m,n) = y \left( \delta_t + c_2 \frac{y z^y}{1 - z^y} - \frac{y z^y}{1 - z^y} \right).
\]

To simplify the expression, let us introduce the following constants. Let \( c_1 = \frac{N q c}{(1 - p_e^m)^{\pi_m^*}} \), \( c_2 = \frac{\delta_t}{1 - p_e} \) and \( c_3 = \frac{\delta_t + p_e \delta_z}{1 - p_e} \). Then, we have:

\[
c_1 \geq n(c_2 + c_3) + \frac{(m - n)y}{1 - z} \left[ \delta_t + c_3 \frac{y z^y}{1 - z^y} - \frac{y z^y}{1 - z^y} \right].
\]

Simplifying, we get:

\[
f(m,n) \geq y \left( \delta_t + c_2 \frac{y z^y}{1 - z^y} - \frac{y z^y}{1 - z^y} \right).
\]

Using equations (24) and (25), we have:

\[
c_1 \geq n(c_2 + c_3) + (m - n)f(m,n)
\]

\[
\Rightarrow mf(m,n) \leq c_1 - n(c_2 + c_3) + nf(m,n)
\]

\[
m \leq n + \frac{c_1 - n(c_2 + c_3)}{f(m,n)}
\]

Since the energy gained in the system is the same for both the CTP and the TTP algorithms, and since the CTP algorithm employs a larger threshold during the On periods and a relatively smaller threshold during the Off periods, the
threshold \( n \) must be lower than the energy-balancing TTP threshold \( m_{\text{TTP}}^{\text{EB}} \) given by equation (14) to satisfy energy balance during a renewal interval. Based upon the above analysis, we have the following result:

Lemma 4: Any energy-balancing CTP pair \((m,n)\) must satisfy the following inequalities:

\[
1. \quad n \leq m_{\text{TTP}}^{\text{EB}} = \frac{Nqc}{\delta_1 + \delta_2 \pi^m} p_d
\]

and

\[
2. \quad n \leq m \leq n + \frac{c_1 - n(c_1 + c_2)}{U(n)(\delta_1 + c_1) - (\delta_2 + p_d \delta_2)},
\]

where \( c_1 = \frac{Nqc}{(1 - p_{\text{off}}^m)} \pi^m \), \( c_2 = \frac{\delta_1}{1 - p_{\text{off}}^m} \) and \( c_3 = \frac{\delta_1 + p_d \delta_2}{1 - p_{\text{off}}^m} \).

Next, we present simulation results for TTP and CTP algorithms.

### 4.3 Simulation results

We evaluate the performance of various activation algorithms using discrete-event simulation of the sensor system. The simulation code is written in C programming language. The system parameters used are \( N = 16 \), \( q = 0.25 \), \( c = 2 \), \( \delta_1 = 1 \), \( \delta_2 = 4 \), \( p_d = 0.5 \) and \( K = 2000 \). Note that the parameters are chosen such that the average recharge rate of a sensor is lower than its discharge rate in the active state. Also, the event detection and transmission cost \( (\delta_1) \) is modelled to be higher than the activation cost \( (\delta_2) \). Experiments are performed for a range of detection probability \( (0.2 \leq p_d \leq 0.6) \), correlation probabilities \( (0.5 \leq p_{\text{off}}^m, p_{\text{off}}^c \leq 0.99) \) and the ratio of event detection versus activation cost \( (1 \leq \beta \leq 28) \). Although results are presented for \( N = 16 \) sensors, we observe similar performance trends for other values of \( N \) as well. The utility function used is \( U(n) = 1 - (1 - p_d)^n \).

Figure 6 depicts the performance of the TTP algorithm with \( p_{\text{off}}^m = 0.5 \) and \( p_{\text{off}}^c = 0.5 \) (no temporal correlations). \( U^* \) denotes the upper bound to maximum achievable performance given by Lemma 2. \( \Pi_{\text{TTP}} \) corresponds to the performance of the TTP algorithm at various values of the threshold parameter. From equation (14), the energy-balancing threshold \( m_{\text{TTP}}^{\text{EB}} \) is 4. Since \( p_{\text{off}}^m = p_{\text{off}}^c = 0.5 \), \( \Pi_{\text{TTP}}^{\text{EB}} \) achieves optimal performance in this case, as suggested by Corollary 1.

Next, we introduce temporal correlations in the application-specific event phenomena with \( p_{\text{off}}^m = p_{\text{off}}^c = x, \forall x \in \{0.6, 0.7, 0.8, 0.9, 0.99\} \). Figure 7 depicts the performance of TTP algorithm at various values of threshold. In all these cases, \( m_{\text{TTP}}^{\text{EB}} = 4 \). We observe that the best TTP performance is obtained at \( m = m_{\text{TTP}}^{\text{EB}} \) and satisfies the performance bound given by Lemma 3. In addition, we observe that as the degree of temporal correlations increases, the ratio of TTP algorithm performance to \( U^* \) decreases at all values of threshold. The decrease in performance is more visible at higher threshold values. Thus, an increase in the degree of temporal correlations worsens system performance under the TTP algorithm.

Figure 8 depicts the performance of \( \Pi_{\text{TTP}}^{\text{EB}} \) algorithm for various values of detection probability \( p_d \) with \( p_{\text{off}}^m = p_{\text{off}}^c = 0.9 \). Note that the upper bound \( U^* \), as well as the energy-balancing threshold \( m_{\text{TTP}}^{\text{EB}} \), varies with \( p_d \). The energy-balancing TTP threshold \( m_{\text{TTP}}^{\text{EB}} \) is not an integer in these scenarios, and the performance for the nearest-rounded integer value is depicted in the figure. As \( p_d \) increases, the energy-balancing threshold \( m_{\text{TTP}}^{\text{EB}} \) decreases, while the performance of \( \Pi_{\text{TTP}}^{\text{EB}} \) approaches the upper bound, as suggested by Lemma 3.

Figure 9 shows the performance of \( \Pi_{\text{TTP}}^{\text{EB}} \) algorithm for various values of \( \beta \) with \( p_{\text{off}}^m = p_{\text{off}}^c = 0.6 \) and \( p_d = 0.5 \). The threshold \( m_{\text{TTP}}^{\text{EB}} \) decreases with an increase in \( \beta \). However, for all values of \( \beta \), \( \Pi_{\text{TTP}}^{\text{EB}} \) achieves performance >96% of maximum achievable performance (given by \( U^* \)).
Figure 8 $\Pi_{TP}^{EB}$ performance with varying detection probability (see online version for colours)

Figure 9 $\Pi_{TP}^{EB}$ performance with varying $\beta$ (see online version for colours)

Table 1 Performance for CTP threshold pairs

<table>
<thead>
<tr>
<th>$(m,n)$</th>
<th>$m=5$</th>
<th>$m=6$</th>
<th>$m=7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n=3$</td>
<td>0.779666</td>
<td>0.823814</td>
<td>0.837527</td>
</tr>
<tr>
<td>$n=4$</td>
<td>0.812619</td>
<td>0.842962</td>
<td>0.817212</td>
</tr>
<tr>
<td>$n=5$</td>
<td>0.831930</td>
<td>0.810727</td>
<td>0.791797</td>
</tr>
</tbody>
</table>

Note: The optimal performance is in bold. $\Pi_{TP}^{EB}$ performance is presented in italics.

From Lemma 4, when $n=4$, energy-balancing pair $(m,n)$ should satisfy: $4 \leq m \leq 6.1$, and when $n=3$, energy-balancing pair $(m,n)$ should satisfy: $3 \leq m \leq 7.96$. Both the $(m,n)$ threshold pairs $(6,4)$ and $(7,3)$ satisfy the conditions outlined in Lemma 4. Hence, while evaluating CTP algorithms, we should focus on the range of thresholds $(m,n)$ as given in Lemma 4. This fact also suggests that the energy-balancing CTP threshold pairs perform better than other CTP threshold pairs.

Figure 10 depicts the performance of various CTP algorithms with $p_{cm}^0 = p_{c}^{off} = 0.9$ and greater detection probability $p_{d} = 0.5$. In this case, $\Pi_{TP}^{EB}$ corresponds to $m=n=4$. We observe that the best CTP performance is achieved for $m=4$ and $n=4$, and hence $\Pi_{TP}^{EB}$ achieves the best performance in this case. The CTP performance trends for this scenario are depicted in more detail in Figure 12. In Figure 12a, the threshold $n$ in the pair $(m,n)$ is kept fixed, and $m$ is varied from 1 to 16 on the x-axis. Similarly, in Figure 12b, the threshold $m$ is kept fixed and $n$ is varied. The two figures correspond to two different 2D projections (cross-sections) of the 3D performance plot depicted in Figure 11. For the fixed threshold of $n$, the value of $m$ corresponding to peak performance decreases as $n$ is increased. Similar behaviour is observed for the fixed threshold of $m$ as well. We observe that for fixed threshold $n=4$, the CTP performance increases with an increase in $m$ from 1 to 4, and later decreases as $m$ is increased further. Similar performance trend is observed for fixed threshold $m=4$, and for other values of $m$ and $n$.

The performance of CTP algorithm with $(m,n)$ pairs $(5,2)$ and $(4,3)$ is also quite close to the best achievable performance (however the performance is slightly lower than that of $\Pi_{TP}^{EB}$). From Lemma 4, the range for energy-balancing threshold pair $(m,n)$ is given by:

- $2 \leq m \leq 5.95$ when $n=2$,
- $3 \leq m \leq 4.66$ when $n=3$ and
- $4 \leq m \leq 4$ when $n=4$.
Note that there is no energy-balancing threshold pair \((m, n)\) such that \(n > 4\). We observe that all the threshold pairs which achieve performance close to optimal belong in the range for energy-balancing threshold pair derived in Lemma 4.

Note that in all the above scenarios, the peak performance of TTP algorithm is achieved at the threshold of \(m_{\text{TTP}}^{EB}\) given by equation (14). Also, the performance improvement, if any, using a CTP algorithm over that of \(\Pi_{\text{TTP}}^{EB}\) is observed to be \(<2\%\) in typical scenarios. In addition, it would be easier to employ a time-invariant threshold of \(m\) when compared with a time-varying threshold pair \((m, n)\) in real sensor network deployments.

Thus, the TTP algorithm with an energy-balancing threshold is suitable in most practical scenarios. Note that sometimes, the threshold \(m_{\text{TTP}}^{EB}\) given by equation (14) might not be an integer value. In such cases, a probabilistic activation scheduling could be used. For instance, let \(I_i\) be an integer such that \(I_i < m_{\text{TTP}}^{EB} < I_i + 1\). Then, in each time slot, a threshold of \(I_i\) could be chosen with probability \(p\) and a threshold of \(I_i + 1\) could be chosen with probability \((1 - p)\) such that the expected threshold (and hence the steady-state time average threshold) equals \(m_{\text{TTP}}^{EB}\).

**Run-time statistics:** It is worth noting that it is not possible to keep all the sensors active, particularly when a large threshold is applied, due to the energy constraints. In the scenario corresponding to Figure 10, we observe that when the energy-balancing TTP threshold of 5 is applied, five sensors are active for 99.99% of the time. However, when a TTP threshold of 10 is applied, the fraction of time \(i\) sensors that are active for \(0 \leq i \leq 10\) is depicted in Figure 13(a).

Thus, employing large threshold only attempts to activate a large number of sensors, if possible. Indeed this explains why the performance curves are not concave for threshold values in Figures 6, 7 and 12. We also observe the energy levels of an arbitrarily chosen sensor over a period of time. For \(\Pi_{\text{TTP}}^{EB}\), the sensor’s energy level is \(\geq 20\%\) of bucket size \(K\) for more than 92% of the time, as shown in Figure 13(b).

On the other hand, with a TTP threshold of 10, the sensor’s energy level is observed to be \(<20\%\) of bucket size \(K\) for 99.97% of the time, resulting in the behaviour shown in Figure 13(a). Similarly, Figures 13(c) and 13(d) depict the run-time statistics in terms of number of active sensors for CTP thresholds of \((6, 4)\) and \((8, 6)\), respectively. In both these cases, the sensor’s energy level is \(<20\%\) of bucket size \(K\) for more than 98% of the time. However, the targeted threshold pair of \((6, 4)\) is achieved more often than the pair \((8, 6)\). For the energy-balancing CTP pair \((6, 4)\), using Figure 5 and equation (18), the threshold of 6 gets applied 42.86% of the time, while a threshold of 4 gets applied 57.14% of the time. From Figure 13(c), these targeted thresholds are achieved almost always. For the CTP pair \((8, 6)\), the targeted thresholds are not achieved for a large fraction of the time, as shown in Figure 13(d). Note that even though the pair \((6, 4)\) lies in the range of energy-balancing threshold pair given by Lemma 4, the sensors spend most of their time with insufficient energy. This suggests that \(\Pi_{\text{TTP}}^{EB}\) maintains a more stable energy level at the sensors than the CTP algorithm.

**Figure 11** CTP performance with large detection probability (see online version for colours)

**Figure 12** CTP performance trends – energy-balancing CTP threshold pairs \((m, n)\) achieve near-optimal performance: (a) fixed threshold \(n\) and (b) fixed threshold \(m\) (see online version for colours)
Figure 13 Run-time statistics in terms of number of active sensors and sensors’ energy levels at steady-state: (a) under a targeted TTP threshold of 10, (b) under energy-balancing CTP threshold pair (6, 4) and (d) under CTP threshold pair (8, 6) (see online version for colours)

Implementation mechanism: We briefly describe a simple mechanism which could be used to implement the threshold policies (TTP or CTP) in a distributed manner. At the beginning of every time slot, each sensor exchanges hello message with all other nodes. Each hello message contains the sender node’s id \( l ((l \in [1\ldots N])) \) and its current energy level \( E_l \). Sensor with id \( l \) could transmit in mini-slot \( l \) to avoid channel contention. After all the messages have been exchanged, each sensor sorts the energy levels of all nodes in descending order and checks whether its energy level is among the top \( m \) energy levels in the system. If yes (no), the sensor activates (deactivates) itself during this time slot. Here, \( m \) is the targeted threshold and corresponds to \( m_{TTP} \) for TTP algorithm. In the case of CTP algorithm, the hello message also contains a bit to indicate whether the sender detected an event during the previous time slot if it was active. Using the bits received from all sensors, and based upon the decision rule for event detection (Examples 1 and 2 in Section 2.3), the sensors decide whether an event was detected in the system. If an event was detected, the sensors target a threshold of \( m \) during this time slot, otherwise a threshold of \( n \) is targeted.

5 Related work

Zhao and Ye (2008, 2009) consider the problem of quickest detection in multiple On/Off processes, and using a decision-theoretic framework, they show that the joint switching and detection rule having a threshold structure achieves optimal performance under a mild condition. Here, the switching threshold and the detection threshold are chosen appropriately based upon the channel conditions and the desired detection reliability. Niyato, Hossain and Fallahi (2007) propose probabilistic sleep and wake-up strategies for a single solar-powered sensor node, where the objective is to reduce packet-dropping and packet-blocking probabilities and study their performance as well as the choice of optimal parameters using a game theoretic approach. Lin et al. (2005) present an online routing algorithm in multi-hop wireless networks with renewable energy sources. Here, each node in the network is assumed to have the knowledge of its short-term energy replenishment schedule, and the proposed algorithm is shown to achieve a competitive ratio which is asymptotically optimal with respect to the number of network nodes.

Banerjee et al. (2007), Banerjee and Kherani (2007), Gatzianas et al. (2008) and Niyato, Hossain and Fallahi (2007) have considered the dynamic activation question in the context of energy-harvesting sensor systems. Banerjee et al. (2007) consider the single sensor, and by modelling it as a closed three-queue system, they obtain Norton’s equivalent of the system to evaluate the structure of the optimal-rate control policy. Banerjee and Kherani (2007) consider the sensor activation problem under a sensor energy model similar to Kar et al. (2006), and demonstrate the optimality of threshold-based policies for a broad class of utility functions and state dynamics. Gatzianas et al. (2008) consider the resource allocation problem in the presence of rechargeable nodes,
propose a policy which decouples admission control and power allocation decisions and show that it achieves asymptotically optimal performance for sufficiently large battery capacity to maximum power ratio.

Activation scheduling in a system of rechargeable sensors has also been considered previously in Kar et al. (2006) and Jaggi et al. (2008). Kar et al. (2006) consider a sensor energy model wherein each sensor could be activated only upon complete recharge of its battery, and can be deactivated only upon complete discharge. Spatial correlation is introduced in the recharge and discharge intervals of the collocated sensors and an energy-balancing threshold-based activation policy is designed which is robust to the presence of correlations and achieves performance greater than 75% of the optimal performance. Jaggi et al. (2008) consider sensor systems where individual sensors could be activated or deactivated at all energy levels. Spatial correlation is introduced in the recharge and discharge processes of collocated sensors and an energy-balancing threshold-based activation policy is designed which is robust to the presence of correlations and achieves asymptotically optimal performance with respect to the sensor energy bucket size.

This paper differs from the above in that it focuses on the uncertainty modelling of the application-specific event phenomena itself instead of introducing correlations in the discharge processes (or intervals) of collocated sensor nodes. In addition, this paper focuses on the temporally correlated nature of event phenomena, compared with spatial correlation specific treatment in Jaggi et al. (2008). Recently, Jaggi et al. (2009) have considered such modelling of event phenomena for a single-sensor system and have designed algorithms which perform close to optimal. In this paper, however, we consider sensor systems with multiple sensor nodes and focus on designing algorithms to solve the multi-sensor activation question under temporally correlated event phenomena. In the single-sensor scenario, the decision of the sensor only includes when to activate or deactivate and for how long, whereas in the case of multi-sensor systems, the decision question is a joint decision among the sensor nodes. The sensors make a collective decision to determine how many sensors to activate at any time, and hence the results in Jaggi et al. (2009) do not simply extend to this scenario. In addition, this paper explicitly considers a probability of detection for each sensor node, in contrast with Jaggi et al. (2009), where the sensor’s detection probability is implicitly assumed to be 1.

6 Summary and conclusions

We have considered the multi-sensor activation question for a renewable-energy-based sensor system in the presence of temporally correlated event phenomena. Particularly, we focused on threshold-based activation policies and derived performance bounds for TTP, which employs a constant, energy-balancing threshold in the system. Assuming sufficiently large energy bucket size $K$, the proposed TTP algorithm achieves optimal performance with no temporal correlations and near-optimal performance in the presence of temporal correlations in the event phenomena. We also analysed the performance of CTP algorithms, which employ a time-varying threshold in the system, and derived feasible ranges for energy-balancing CTP threshold pairs. The TTP algorithm is simpler, involves minimum state maintenance overhead and would be suitable to deploy in most practical scenarios. Although, in general, TTP algorithm can lead to suboptimal performance, the performance improvement gained by employing CTP algorithm is observed to be insignificant in the scenarios considered.

We have considered a first-order correlation model to model the temporal correlations in the event phenomena. One of the future directions would be to explore the extension of these results to higher-order correlation models, possibly with multiple event processes, and to show the optimality (or near-optimality) of threshold policies in such general scenarios. Another direction of future research includes the consideration of false alarms generated by the sensors during the Off periods and finding an activation policy which maximises the detection of genuine events, while keeping the false alarm rate low or within a specified bound.

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References


Notes

1. This paper extends the results that appeared in WiOpt 2009, Seoul (Jaggi and Kar, 2009).

2. Even though we have considered a Bernoulli recharge process, the results in this paper could easily be extended to the case of correlated recharge process, since we only consider the average recharge rate in the analysis (and in most cases, over an infinite horizon).

3. Note that we consider a single-event process with temporal correlations in this paper. The consideration of multiple-event processes, with possible correlations amongst them, is beyond the scope of this work and could be considered for future extensions.
Appendix A: Effect of boundary conditions on performance with energy-balancing threshold

We show that for an energy-balancing threshold policy, the probability that the employed threshold is not met is of the order $o\left(\frac{1}{K}\right)$, where $K$ is the sensor energy bucket size. For simplicity of exposition, we perform this analysis for the TTP activation algorithm $\Pi^{12}_{\text{TTP}}$. Similar results are expected to hold for an energy-balancing CTP algorithm as well.

Consider an On/Off cycle of length $T_c$ for the sensor system operating under the energy-balancing TTP algorithm $\Pi^{12}_{\text{TTP}}$, with threshold parameter $m = m^{12}_{\text{TTP}}$. Let $R$ denote the number of energy quanta received by the sensor system through recharge, and $D$ denote the number of energy quanta spent by the sensor system during this cycle. From equations (12)) and (13), we have:

$$E[R] = \frac{Nqc}{\left(1-p^K\right)}\pi^m$$

$$E[D] = \frac{m\delta + np\delta}{\left(1-p^K\right)}\pi^m$$

Let $T_1, T_2, \ldots, T_M$ denote the lengths of $M$ consecutive On/Off cycles. Let $(R_1, D_1), \ldots, (R_M, D_M)$ denote the number of energy quanta received and spent, respectively, during these $M$ cycles. Note that $R_i, 1 \leq i \leq M$ are i.i.d. (independent and identically distributed) random variables. Similarly, $D_i, 1 \leq i \leq M$ are i.i.d. random variables. Also, $R_i$ and $D_i$ are independent for all $i, 1 \leq i \leq M$. Let us define $M$ random variables, $Z_i = D_i - R_i, 1 \leq i \leq M$.

Let $Z = \sum_{i=1}^{M} Z_i \pi^m$. Note that:

$$E[Z] = 0; \quad \text{Var}[Z] = \frac{\text{Var}[D] + \text{Var}[R]}{M}.$$  

From Chebyshev’s inequality (Papoulis and Pillai, 2002), we have:

$$\Pr\left[ |Z - E[Z]| \geq \varepsilon \right] \leq \frac{\text{Var}[Z]}{\varepsilon^2}.$$  

This implies:

$$\Pr\left[ |Z| \geq \varepsilon \right] \leq \frac{\text{Var}[D] + \text{Var}[R]}{M\varepsilon^2}. \quad (A1)$$

Let us assume that each of the sensors had an energy level of $\frac{K}{2}$ (or of the order $O(K)$) at the beginning of the first cycle. At the end of $M$th cycle, the threshold of $m$ will not be met if and only if at least $N - m + 1$ sensors have run out of their energy at this time, i.e., only if $\sum_{i=1}^{M} D_i - \sum_{i=1}^{M} R_i > (N - m + 1)O(K)$. The above event is the same as the event $Z > (N - m + 1)O(K)$ or $Z > \frac{(N - m + 1)O(K)}{M}$. Using equation (A1), we obtain:

$$\Pr\left[ |Z| \geq \frac{(N - m + 1)O(K)}{M} \right] \leq \frac{(\text{Var}[D] + \text{Var}[R])}{(N - m + 1)^2O(K^2)}. \quad (A2)$$

Let us assume that for some $M$, at the end of $M$th cycle, the threshold of $m$ is not met. Let $t$ denote the number of time slots after the $M$th cycle for which the desired threshold is not met. We have: $E[t]\leq \frac{(\delta + \delta_t)}{qc}$. The total number of events expected to occur in $M$ cycles is given by $ME[T]_m\pi^m$. Therefore, the fractional loss in performance for the activation policy $\Pi^{12}_{\text{TTP}}$ is

$$\frac{(\delta + \delta_t)}{qcME[T]_m\pi^m} \leq \frac{(\delta + \delta_t)}{qcME[T]_m\pi^m} \leq \frac{(\delta + \delta_t)}{qcME[T]_m\pi^m}. \quad (A3)$$

from equation (A2). Assuming $K \gg N$, the expected fractional loss in utility of $\Pi^{12}_{\text{TTP}}$, denoted $\tau$, satisfies:

$$\tau \leq \frac{(\delta + \delta_t)}{qcME[T]_m\pi^m} \leq \frac{(\delta + \delta_t)}{qcME[T]_m\pi^m} \leq \frac{(\delta + \delta_t)}{qcME[T]_m\pi^m}. \quad (A3)$$

Since equation (A3) holds for all $M$, it follows that if $K$ is sufficiently large, the energy-balancing threshold of $m = m^{12}_{\text{TTP}}$ will almost always be met. The performance achieved by $\Pi^{12}_{\text{TTP}}$ is

$$\geq \frac{U(m^{12}_{\text{TTP}}) - (\delta + \delta_t)}{\pi^m}, \quad \text{from equation (15).}$$