Robust Threshold based Sensor Activation Policies under Spatial Correlation

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Abstract—We consider a system of rechargeable sensor nodes deployed redundantly in the region of interest. Active sensor nodes are subject to a discharge process which depends on the occurrence of events in the system. All sensor nodes experience a continuous recharge of their battery energies, however at a considerably lower rate than of discharge. The decision problem is to find the optimal activation policy for the sensor nodes which would maximize the aggregate system utility. We model our rechargeable sensor system as a system of finite-buffer queues and show the existence of a simple threshold activation policy that achieves near optimal performance. Our results hold both in presence as well as absence of correlation in the discharge and/or recharge processes, thereby showing the robustness of such policies with respect to the degree of correlation in the system.

I. INTRODUCTION AND MODEL

Advances in sensor network technology enable sensor nodes with renewable energy sources, e.g. rechargeable batteries, to be deployed redundantly in the region of interest to optimize a global system objective. One such objective is the quality of coverage provided in the region. Utilizing the redundancy of sensing devices in the system effectively so as to provide a better quality of coverage in the region, motivates the need to design efficient sensor node activation schedules in the system.

We consider such a system of rechargeable sensor nodes deployed redundantly in the region of interest and analyze its performance under threshold activation policies. A sensor node is modeled as a finite buffer Markovian queue where recharge quanta arrive at the sensor node with buffer (bucket) size $K$, according to a poisson process and the discharge of these quanta takes place according to another poisson process, however at a much larger rate. Moreover, the discharge process acts upon a sensor node only if the node has been activated in the system (refer Figure 1).

Consider a system consisting of $N$ sensor nodes providing coverage to a common region of interest. The system performance is measured using a continuous, non-decreasing, strictly concave utility function $U(n_A)$, where $n_A$ is the number of active sensors at any time. Also $U(0) = 0$. For instance, if the detection probability of an event at a sensor is $p_d$, then the probability that the event gets detected with $n_A$ active sensors in the system is given by $U(n_A) = 1 - (1 - p_d)^{n_A}$. Now, the time average utility of the system for an activation policy $P$ is given by $U_T(P) = \lim_{T \to \infty} \frac{1}{T} \int_0^T U(n_A(P,t))dt$. The goal is to find a node activation policy $P$ for which $U_T(P)$ is maximized for given system parameters (i.e. the recharge and discharge (event) processes, $p_d$ and $N$).

We focus on the class of threshold activation policies, where an available sensor (i.e. one with non-zero energy) is activated if the number of active sensors in the system is less than a threshold $m$. Thus a threshold activation policy $P$ with a threshold of $m$ tries to maintain the number of active sensors in the system as close to $m$ as possible (however never exceeding $m$).

Once a sensor gets activated, it remains active until it gets completely discharged. Once completely discharged the sensor stays inactive until it receives a recharge quantum, at which time the sensor moves to ready state. A sensor in the ready state gets activated as soon as the number of active sensors in the systems falls below the specified threshold $m$. Such a system can be represented as a system of $N$ queues with $m$ exponential servers, where each queue represents a sensor node. Each server selects a queue with non-zero quanta and serves it until the queue becomes empty (i.e. the sensor gets discharged completely), after which the server tries to select another non-empty queue which is not being served (if available), representing the activation of a ready sensor node and so on. When the discharge and recharge processes at the different sensors are independent, this system of queues is referred to as the Independent Discharge-Recharge (IDR) model [2] (refer Figure 2). The recharge and discharge rates are given by $\lambda$ and $\mu$ respectively, with $\rho = \frac{\lambda}{\mu} \leq 1$, since recharge rate is typically lower than the discharge rate.

In practical scenarios, due to spatial correlation in sensor node placements, the discharge and/or recharge processes at different sensors could be correlated. A perfect correlation in the discharge and the recharge processes at the sensors, along with threshold-based activation leads to batch activation and deactivation of these sensors. In this case, with a threshold of $m$, the system can be represented as a polling system with $cl (= \frac{m}{\lambda})$ queues (assuming $c$ is an integer) and one exponential server, where each of these queues represents a group of $m$ sensors. Again, the server selects one of the non-empty queues and serves it until the queue becomes empty, at which time the server selects another non-empty queue and so on. This system of queues is referred to as the Correlated Discharge-Recharge (CDR) model [2] (refer Figure 3). Note that in real world, the amount of correlation would lie somewhere between perfect correlation and complete independence, and

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hence considering only these two extreme scenarios may not be sufficient. However, the results we obtain below suggest that the effect of correlation does not alter the optimal performance of threshold activation policies.

The time average utility of the sensor system can be expressed in terms of the time average (or steady-state) utilization of the corresponding queuing system. For the IDR model, $i$ servers being busy at a time corresponds to a utility of $U(i)$ being achieved in the sensor system. Similarly, for the CDR model, the server being busy corresponds to a utility of $U(m)$ being achieved in the sensor system. Thus the time average utility of the sensor system can be computed using the steady-state utilization of the corresponding queuing system.

The problem of rechargeable sensor activation has been studied in [1], [2]. Kar et al. [1] consider a restrictive system model wherein sensor nodes can only be activated when fully recharged and show that for a particular choice of threshold, the system achieves a utility no less than $\frac{3}{4}$ of the maximum achievable utility. Jaggi et al. [2] consider nodes which could be activated as long as they have a non-zero energy level and show that for a particular choice of threshold $2\gamma$, the system achieves asymptotically optimal performance with respect to the buffer size $K$. However, the results in [2] hold only when a particular group Longest Undischarged-time First (g-LUF) scheduling discipline is employed in the above queuing systems. In fact, the g-LUF scheduling is restrictive in nature. This is because using LUF scheduling, the server serves only one quantum at a time from the queue, after which it selects another queue to serve. Such a scheduling implies that a sensor (or a group of sensors) gets activated for a very small amount of time (discharge time of one quantum) before being deactivated, implementing which may be costly, or even infeasible, in practice.

We address these limitations by considering a general class of work-conserving scheduling disciplines, wherein a server remains busy as long as there exists a non-empty queue which is not being served. Such disciplines capture most of the practical activation scenarios including activation until complete discharge. We enhance the results in [2] in several significant ways. We generalize the performance results in [2] to the class of work-conserving scheduling disciplines. We consider all possible combinations of correlation across discharge and/or recharge processes among sensor nodes in the system and show the robustness of threshold activation policies under all such models. The Independent Discharge Correlated Recharge (IDCR) model is similar to the IDR model, however with perfectly correlated recharge among the sensors. Similarly, the Correlated Discharge Independent Recharge (CDIR) model is
similar to the CDR model, however the recharge processes at
the different sensor nodes are mutually independent. Figures
8 and 9 depict the queueing diagrams of the IDR and CDIR
models respectively. We also provide a special treatment to
the threshold of \( N \), which leads to better insights into the
problem.

II. MAIN RESULTS

Consider a system comprising of \( N \) sensor nodes. Each of
the sensors is subject to a recharge process which is poisson
with rate \( \lambda \). When activated, a sensor is also subject to
a discharge process which is poisson with rate \( \mu \) (refer Figure
1). For simplicity of analysis, we assume that \( \gamma (= \frac{\lambda}{\mu}) \) is
an integer and a factor of \( N \). First we derive some analytical
results for the maximum threshold of \( N \) for the various
models introduced earlier (i.e. for IDR, IDCIR, CDR and CDIR
models). Next we show that a threshold activation policy, wi th
a particular choice of threshold (\( \gamma \)), is asymptotically optimal
with respect to the bucket size \( K \) for all these models. These
results hold for any work-conserving scheduling discipline
employed in the system.

Lemma 1: The Time Average Utility achieved by any node
activation policy \( P \), \( U_T(P) \), is upper bounded as

\[
U_T(P) \leq U\left(\frac{N}{\gamma}\right)
\]

(1)

The proof of this lemma can be found in [2]. This result
holds for all the different system models.

A. Threshold of \( N \)

The activation decision for a threshold of \( N \) is trivial, and
effectively a sensor gets activated as soon as it has a non-zero
energy level. Recall, \( \rho = \frac{\lambda}{\mu} = \frac{1}{\gamma} \leq 1 \). Let \( U_T(m) \) denote the
time-average utility achieved by a threshold activation policy
with threshold parameter \( m \).

Let us first consider a system with a single sensor node
\((N = 1)\), having a charge bucket size of \( K \). The activation
policy has a threshold of \( N = 1 \). The sensor node behaves
as a \( M/M/1/K \) queue, providing for a utility of \( U(1) \) as long
as the charge bucket is non-empty i.e. there exists at least one
quantum in the system. Let \( x \) denote the probability that there
are no quanta in the system. Then,

\[
x = \frac{1 - \rho}{1 - \rho K + 1}
\]

(2)

when \( \rho < 1 \) and \( x = \frac{1}{K+1} \) when \( \rho = 1 \). The time average
utility of the system is given by

\[
U_T(1) = (1 - x)U(1) = \rho\left(1 - \frac{\rho^K}{1 - \rho K + 1}\right)U(1)
\]

(3)

when \( \rho < 1 \), and by

\[
U_T(1) = (1 - x)U(1) = \frac{K}{K + 1}U(1)
\]

(4)

when \( \rho = 1 \). Note here that for \( \rho = 1 \), a threshold of
\( m = \frac{N}{\gamma} = 1 \) is asymptotically optimal with respect to \( K \), since
the upper bound to achievable utility is \( U(1) \) (from Lemma
1). We also observe that for \( \rho < 1 \) and for considerably large
buffer size \( K \), \( U_T(1) \rightarrow \rho U(1) \) (from (3)). Next we consider
the case of general \( N (N > 1) \) and show that both the above
facts hold true in this case as well. We consider all the four
models presented earlier.

1) IDR Model: Each of the sensors behave as independent
\( M/M/1/K \) queues providing for utility as long as their charge
bucket is non-empty. The probability that the charge bucket is
empty for any of the sensor nodes is given by

\[
Q(i) = \binom{N}{i}(1 - x)^i x^{(N - i)}
\]

(5)

The steady-state time average utility achieved in the system
is given by

\[
U_T(N) = \sum_{i=1}^{N} Q(i)U(i)
\]

(6)

Expanding, and using the fact \( \frac{U(k)}{U(k)} \geq \frac{U(N)}{U(N)} \forall k \in
[1,...,N - 1] \) which follows from the concavity and non
decreasing nature of \( U \), and the fact that \( U(0) = 0 \), we have,

\[
U_T(N) \geq (1 - x)U(N)\left[\sum_{i=0}^{N-1} \binom{N-1}{i}(1 - x)^i x^{(N - i)}\right]
\]

\[
= (1 - x)U(N)
\]

\[
= \rho\left(1 - \frac{\rho^K}{1 - \rho K + 1}\right)U(N)
\]

(7)

when \( \rho < 1 \) and \( U_T(N) \geq (1 - x)U(N) = \frac{K}{K+1}U(N) \)
when \( \rho = 1 \).

2) CDR Model: When the discharge and recharge processes
at all the sensors are perfectly correlated, all the sensors get
activated (and deactivated) at the same time. This system can
be represented as a \( M/M/1/K \) queue providing a utility of
\( U(N) \) in the system as long as the queue has non-zero quanta.
Thus,

\[
U_T(N) = (1 - x)U(N) = \rho\left(1 - \frac{\rho^K}{1 - \rho K + 1}\right)U(N)
\]

(8)

when \( \rho < 1 \) and \( U_T(N) = \frac{K}{K+1}U(N) \) when \( \rho = 1 \).

For the CDR (and IDCIR) model, since the recharge (discharge)
processes are independent across the sensor nodes, an
analysis similar to that used for the IDR model can be applied
to show similar results.

Thus, the time average utility for a threshold of \( N \) for all
the above models is lower bounded by \((1 - x)U(N)\). Since
\( K \) represents the granularity of the energy discharge/recharge
and can be assumed to be sufficiently large, considering the
asymptotic behavior as \( K \rightarrow \infty \), we have the following result.
The time average utility attained by a threshold policy as $K \rightarrow \infty$, is referred below as the asymptotic time average utility, and is denoted by $U_T^*$.

**Lemma 2**: The asymptotic time average utility for a threshold activation policy with a threshold of $m = N$ is no less than $\rho$ times $U(N)$ i.e.

$$
\frac{U(N)}{\rho} \leq U_T^*(N) \leq U(N) \quad (9)
$$

Note that the lower bound derived here is tight, since for the CDR model the lower bound $(\frac{U(N)}{\rho})$ for a threshold of $m = N$ is actually achieved. It is also interesting to observe that as $\gamma$ approaches one i.e. the discharge and recharge rates become equal, the lower bound approaches the upper bound and the threshold of $N$ is the optimal threshold.

In cases where $\gamma \gg 1$, the lower bound given by Lemma 2 could be loose for the IDR model and hence we derive a different lower bound in terms of fraction of maximum achievable utility.

**Lemma 3**: The asymptotic time average utility for a threshold activation policy with a threshold of $m = N$ for the IDR model is lower bounded as

$$
U_T^*(N) \geq \frac{1}{2} U(N) \quad (10)
$$

**Proof:**

Let $w = \frac{N}{\gamma}$, such that $1 \leq w \leq N$. Now from (6),

$$
U_T(N) = \sum_{i=1}^{N} Q(i)U(i) = \sum_{i=1}^{w-1} Q(i)U(i) + \sum_{i=w}^{N} Q(i)U(i)
$$

$$
\geq U(w)\left[ \sum_{i=1}^{w-1} \frac{i}{w}Q(i) + \sum_{i=w}^{N} Q(i) \right]
$$

since $U(i) \geq U(w)\forall i \in [w, ..., N]$ and $\frac{U(i)}{i} \geq \frac{U(w)}{w}\forall i \in [1, ..., w-1]$.

Thus,

$$
\frac{U_T(N)}{U(w)} \geq \sum_{i=1}^{w-1} \frac{i}{N}C_i(1-x)^i \gamma (N-i) + \sum_{i=w}^{N} N C_i (1-x)^i x^{N-i}
$$

$$
= \sum_{i=1}^{w-1} \frac{(N-1)}{N} C_{i-1} (1-x)^{i-1} x^{N-i}(1-x)\gamma
$$

$$
+ \sum_{i=w}^{w-2} C_i (1-x)^i x^{N-i}
$$

$$
= \sum_{i=0}^{w-1} \frac{(N-1)}{N} C_{i-1} (1-x)^{i-1} x^{N-i}(1-x)\gamma
$$

$$
+ \sum_{i=w}^{N} N C_i (1-x)^i x^{N-i}
$$

The right hand side is further simplified by noting that for $0 \leq j < w$,

$$
(N-1)C_j x^{(N-1-j)} (1-x)^j \gamma (1-x) \geq \frac{1}{2} N C_j x^{(N-j)} (1-x)^j
$$

and

$$
N C_x (1-x)^w x^{(N-w)} \geq N C_x (1-x)^x x^{(N-x)}
$$

Note, (11) holds for $K \geq \log_{\gamma} 2$ and (12) holds for $K \geq \log_{\gamma} N$. Note that $K$ is the granularity of the energy bucket and hence can be assumed to be sufficiently large. Thus, we have,

$$
\frac{U_T(N)}{U(w)} \geq \frac{1}{2} \sum_{i=0}^{w-2} N C_i (1-x)^i x^{N-i} + \sum_{i=w}^{N} N C_i (1-x)^i x^{N-i}
$$

$$
= \frac{1}{2} \sum_{i=0}^{w-2} N C_i (1-x)^i x^{N-i} + \frac{1}{2} N C_x (1-x)^w x^{N-w}
$$

$$
+ \frac{1}{2} N C_x (1-x)^w x^{N-w} \geq \sum_{i=w+1}^{N} N C_i (1-x)^i x^{N-i}
$$

$$
= \frac{1}{2} \sum_{i=0}^{w} N C_i (1-x)^i x^{N-i} + \frac{1}{2} \sum_{i=w+1}^{N} N C_i (1-x)^i x^{N-i}
$$

$$
= \frac{1}{2}
$$

**Corollary 1**: The asymptotic time average utility for the IDR model is lower bounded as

$$
U_T^*(N) \geq \max\left(\frac{1}{2} U(N), U_T(N)\right) \quad (13)
$$

**B. Optimality of Threshold Policies**

We analyze the performance of threshold activation policy for a particular choice of chosen threshold and show that such a policy achieves near optimal performance for all the system models described earlier. We show that the policy $P$ with a threshold of $\frac{N}{\gamma}$ is asymptotically optimal with respect to the bucket size $K$, i.e. the time average utility achieved in the system approaches the maximum achievable utility as $K \rightarrow \infty$.

1) IDR model (Independent Discharge Recharge): The sensor system employing a threshold activation policy with a threshold of $m$ is represented using $N$ queues, each with an independent poisson arrival process with rate $\lambda$ and $m$
independent servers whose service times are each exponentially distributed with mean $\frac{1}{\mu}$ (refer Figure 2). Let us call this system $S$. Note that we are considering the threshold $m = \frac{N}{\gamma}$.

Consider any work conserving scheduling discipline $W$. Under the discipline $W$, the server remains busy as long as there is at least one quantum in any of the $N$ queues.

Now, let us constrain this system such that server $j$ ($1 \leq j \leq m$) can only serve queues $(j - 1)c + 1$ to $jc$, where $c = \frac{N}{m} = \gamma$. We refer to this new system as $S'$ (Figure 4). In system $S'$, a server may remain idle even if there exists a non-empty queue which is not being served currently, unlike system $S$. Therefore, the system $S$ has better utilization than the system $S'$.

Now let us superimpose the arrival processes at these groups of $c$ queues to form system $S''$ having $m$ queues, each with a buffer size of $K$. Since the arrivals at the $c$ queues are independent poisson processes with rate $\lambda$, their superposition is also a poisson process with rate $c\lambda$. Thus, the system $S''$ is composed of $m$ independent $M/M/1/K$ queues with arrival rate $c\lambda$ and service rate $\mu$ (refer Figure 5). Since the buffer space in system $S''$ is less than that in the system $S'$, system $S'$ has better utilization than the system $S''$.

In system $S''$, the probability for each of the $m$ queuing systems to be empty is given by $x = \frac{1}{K + 1}$. Therefore, the probability that $i$ out of $m$ queuing systems are empty is given by $mC_i(1-x)^i x^{m-i}$. Using arguments similar to those used in (7), the time average utility of the sensor system corresponding to the queuing system $S''$ is given by,

$$U_T(m) = \sum_{i=1}^{m} mC_i(1-x)^i x^{m-i} U(i)$$

$$\geq (1-x)U(m) = \frac{K}{K+1} U\left(\frac{N}{\gamma}\right)$$

Since the system $S$ has better utilization than the system $S''$, the time average utility for the sensor system corresponding to queuing system $S$ is more than that of the sensor system corresponding to the queuing system $S''$.

**Lemma 4:** The time average utility for a threshold activa-
tion policy with a threshold of \( m = \frac{N}{\gamma} \) for the IDR model is lower bounded as

\[
U_T \left( \frac{N}{\gamma} \right) \geq \frac{K}{K + 1} U \left( \frac{N}{\gamma} \right)
\]  

(15)

2) **CDR model (Correlated Discharge Recharge):** Consider the CDR model in Figure 3 with a threshold of \( m = \frac{N}{\gamma} \). The queuing system is comprised of \( c \) queues, where \( c = \frac{N}{\gamma} = \gamma \) and one exponential server with service rate \( \mu \). Under any work conserving scheduling discipline \( W \), the server remains busy as long as there is at least one quantum in any of the \( c \) queues, thus providing for a utility of \( U \left( \frac{N}{\gamma} \right) \) in the system as long as the server remains busy. Therefore, if the utilization of the server is given by \( \alpha \), the time average utility is given by \( \alpha U \left( \frac{N}{\gamma} \right) \). We shall show that the utilization \( \alpha \) is lower bounded by \( \frac{K}{K + 1} \) for the above choice of threshold i.e. when \( c = \gamma \). We prove this using mathematical induction on \( \gamma \). Let the arrival rate \( \lambda \) be held constant and the service rate \( \mu \) equals \( \gamma \) times \( \lambda \).

**Lemma 5:** The utilization of the queuing system in Figure 3 is at least \( \frac{K}{K + 1} \) when \( c = \gamma \).

Proof:

**Base Case:** Consider the case when \( \gamma = 1 \). Hence \( c = \gamma = 1 \) i.e. there is only one queue with buffer size \( K \). Also, we have \( \mu = \lambda \). Thus we have a \( M/M/1/K \) system with \( \rho = 1 \). The utilization of this system is given by

\[
1 - \pi_0 = \frac{K}{K + 1}
\]  

(16)

**Induction Step:** Let Lemma 5 hold for \( \gamma = i \). That is, when there are \( c = i \) queues with buffer size \( K \) each and \( \mu = i\lambda \), the utilization of the queuing system is at least \( \frac{K}{K + 1} \). Let us call this system \( S \) (Figure 6).

Now consider the case when \( \gamma = i + 1 \). Here we have \( i + 1 \) queues with buffer size \( K \) each and \( \mu = (i + 1)\lambda \). Let us call this queuing system \( S' \) (Figure 7). Note that the sensor system corresponding to the queuing system \( S' \) has \( N + m \) sensors and employs a threshold of \( m \).

Now let us remove one of the buffers from \( S' \) to get a system \( S'' \). The system \( S'' \) has \( i \) queues with buffer size \( K \) and arrival rate \( \lambda \) each, and the service rate of the server is given by \( \mu = (i + 1)\lambda \). Clearly, the system \( S'' \) has better utilization than the system \( S' \). Now let us reduce the service rate of the server in system \( S'' \) from \( \mu = (i + 1)\lambda \) to \( \mu = i\lambda \). Let us call this system \( S''' \). Clearly the system \( S''' \) has better utilization than \( S'' \). Thus system \( S'' \) has better utilization than \( S''' \). Also \( S''' \) is the same as the system \( S \) above (since \( \lambda \) remains the same), for which we assumed the utilization to be at least \( \frac{K}{K + 1} \). Hence the system \( S' \) has utilization at least \( \frac{K}{K + 1} \).

**Lemma 6:** The time average utility for a threshold activation policy with a threshold of \( m = \frac{N}{\gamma} \) for the CDR model is lower bounded as

\[
U_T \left( \frac{N}{\gamma} \right) \geq \frac{K}{K + 1} U \left( \frac{N}{\gamma} \right)
\]  

(17)

3) **IDCR model (Independent Discharge Correlated Recharge):** The IDCR model is represented as a system of \( N \) queues having perfectly correlated poisson arrivals at each queue with rate \( \lambda \), and \( m \) independent servers with service rates \( \mu \) each (refer Figure 8). Each of the queues has a buffer size of \( K \). Note that \( m = \frac{N}{\gamma} \). We shall show that the time average utility of this system is at least \( \frac{K}{K + 1} U \left( \frac{N}{\gamma} \right) \). This is shown by induction over \( \gamma \), in a manner similar to the one used for the CDR model.

For \( \gamma = 1 \), \( m = \frac{N}{\gamma} = N \) and the system reduces to the IDR model with \( N \) queues and \( N \) servers. From (7) with \( \rho = \gamma = 1 \), we have,

\[
U_T \left( \frac{N}{\gamma} \right) = U_T (N) \geq \frac{K}{K + 1} U (N) = \frac{K}{K + 1} U \left( \frac{N}{\gamma} \right)
\]  

(18)
Let this be true for \( \gamma = i \). That is for system \( S' \) with \( N \) queues having perfectly correlated arrivals with rate \( \lambda \) and \( m = \frac{N}{\gamma} \) servers with service rates \( \mu = i\lambda = \frac{N}{\gamma}\lambda \) (refer Figure 10), the time average utility achieved be \( \geq \frac{K}{K+1}U(m) \). 

Now let us consider the system \( S'' \) representing \( \gamma = i+1 \) as a system of \( N + m \) queues having perfectly correlated arrivals with rate \( \lambda \) and \( m = \frac{N+m}{\gamma+1} \) servers with service rates \( \mu = (i+1)\lambda = \frac{N+m}{\gamma+1}\lambda \) (refer Figure 11). Arguing on the same lines as in Lemma 5, system \( S'' \) has better utilization than that of system \( S' \). Thus we have the following result.

**Lemma 7:** The time average utility for a threshold activation policy with a threshold of \( m = \frac{N}{\gamma} \) for the IDCR model is lower bounded as

\[
U_T\left(\frac{N}{\gamma}\right) \geq \frac{K}{K+1}U\left(\frac{N}{\gamma}\right)
\]  

(19)

4) **CDIR model (Correlated Discharge Independent Recharge):** The CDIR model is represented as a system of \( N \) queues having independent poisson arrivals at each queue with rate \( \lambda \), and \( m \) servers with service rates \( \mu \) each (refer Figure 9). Each of the queues has a buffer size of \( K \). Note that \( m = \frac{N}{\gamma} \). We shall show that the time average utility of this system is asymptotically optimal with respect to the buffer size \( K \). This is shown in a manner similar to the one used for the IDR model.

Correlated discharge in the sensor system refers to the fact that the energy quanta at the different sensor nodes which are active simultaneously, get consumed at the same instance of time. For instance, if there are \( i \) active sensors in the system at time \( t \), each of these sensors consume one quantum of energy at some time \( t_1 > t \). Also, the time interval \( t_1 - t \) is exponentially distributed with mean \( \frac{1}{\mu} \).

Let us call the queuing system corresponding to the CDIR model as \( S \). Now, let us constrain this system such that server \( j \) \((1 \leq j \leq m)\) can only serve queues \((j-1)c+1 \) to \( jc \), where \( c = \frac{N}{m} = \gamma \). In addition, the servers serve the queues in the following restricted manner. A server \( j \) is ready to serve if it is currently idle and there exists at least one quantum in any of its corresponding queues i.e. in any of the queues \((j-1)c+1 \) to \( jc \). The servers in the constrained system serve the queues only when all of the servers are ready simultaneously. We refer to this new system as \( S' \), which is similar to the system shown in Figure 4 except the fact that the servers have correlated service times. Since the discharge is correlated, all the servers become idle at the same time and wait for all of them to become ready before servicing again. In system \( S' \), a server may remain idle even if there exists a non-empty queue which is not being served currently, unlike system \( S \). Therefore, the system \( S \) has better utilization than the system \( S' \).

Now let us superimpose the arrival processes at these groups of \( c \) queues in system \( S' \) to form system \( S'' \). Since the arrivals at the \( c \) queues are independent poisson processes with rate \( \lambda \), their superposition is also a poisson process with rate \( c\lambda \). Thus, the system \( S'' \) is composed of \( m \) \( M/M/1/K \) queues each with an arrival rate of \( c\lambda \) and service rate \( \mu \). The servers serve the queues in the same restricted manner as in the system \( S' \), i.e. they wait for all of the servers to become ready before servicing the queues. The system \( S'' \) is similar to the system shown in Figure 5. Since the buffer space in system \( S'' \) is less than that in the system \( S' \), system \( S' \) has better utilization than the system \( S'' \).

In system \( S'' \), the probability for each of the \( m \) queues to be empty is given by \( x \leq \frac{1}{m+1} \). The inequality results due to the fact that the server may have to wait for other servers to be ready before servicing the queue, resulting in more quanta held in queues than in a regular \( M/M/1/K \) queue with \( \rho = 1 \). Therefore the probability that all of the queues are non-empty in the system \( S'' \) is \( \geq \left( \frac{K}{K+1} \right)^m \). When all the servers are non-empty, the servers serve the queues providing for a utility of \( U(m) \) till the time they become idle again. Thus the time average utility provided by the sensor system corresponding to the queuing system \( S'' \) is given by,

\[
U_T(m) \geq \left( \frac{K}{K+1} \right)^m U(m) = \left( \frac{K}{K+1} \right)^m \frac{K}{K+1}U\left(\frac{N}{\gamma}\right)
\]  

(20)

Since the system \( S \) has better utilization than the system \( S'' \),
the time average utility for the sensor system corresponding to queuing system $S$ is more than that of the sensor system corresponding to the queuing system $S''$. Thus we have the following result.

**Lemma 8:** The time average utility for a threshold activation policy with a threshold of $m = \frac{N}{\gamma}$ for the CDIR model is lower bounded as

$$U_T\left(\frac{N}{\gamma}\right) \geq \left(\frac{K}{K+1}\right)^{\gamma} U\left(\frac{N}{\gamma}\right)$$

(21)

**REFERENCES**


