I. INTRODUCTION

A wireless sensor network (WSN) consists of devices (nodes), with sensing and wireless communication capabilities, that are spatially distributed over a sensor field to monitor a set of physical phenomena [11], [23]. The applications of WSNs include environmental monitoring [16], [26], habitat monitoring in remote areas [1], [19], surveillance [7], [21], motion tracking [15], [28] etc. The sensor nodes in a WSN are typically tiny battery powered devices with limited communication and computational capabilities. It is important to conserve energy of the sensor nodes in order to ensure the longevity of the WSN. The most common way of conserving energy is to schedule only a small subset of nodes to be in active state at a given time while other nodes remain in low energy sleep state. The objective of the scheduling scheme is to maximize the lifetime of the sensor network while covering most of the sensor field at all times. At the same time, for proper operation of the sensor network, all the active sensors at any given time must be fully connected.

In recent years there has been an increased interest in rechargeable sensors [9], [8], [13], [14]. In contrast to battery powered sensors, which become useless once they run out of battery, the battery of re-chargeable sensors may be replenished. So theoretically the lifetime of a re-chargeable sensor networks is infinite. However, in most scenarios recharging is a slow process and the time in which a sensor discharges may be much smaller than the time it takes to recharge it. So even in the case of rechargeable sensor networks, there is a need to carefully schedule the sensors between active, sleep and recharge states. If all the charged sensors are allowed to be active at the same time then they may all get discharged simultaneously, leaving the sensor field to be uncovered for a long period of time. As opposed to battery powered sensors, where the aim of scheduling is to maximize the lifetime, the aim of scheduling in the case of rechargeable sensors is to maximize the time average utility in the network, where the utility function would typically reflect the quality of coverage provided. In this paper the utility function considered is the event detection probability.

In the rest of this paper we refer to the sensor networks with rechargeable sensors as rechargeable sensor networks, while the sensor networks with non-rechargeable sensors are referred to as battery-driven sensor networks. It should however be noted that the rechargeable sensors also run on some kind of battery which may be recharged using some renewable source of energy, such as solar, wind, thermal energy etc.

A distributed scheme for scheduling nodes in a sensor network is desirable in order to make it resilient against node failures. Since communication incurs a large energy cost, it is desired that the distributed scheme should require minimal communication. If a distributed scheme requires nodes to exchange large number of messages then the whole purpose of node scheduling, which is to conserve energy, is lost. Also because of battery exhaustion, the sensor network may be divided into disconnected components making it impossible for nodes to exchange messages with other nodes. The requirement of low message exchange overhead makes the development of efficient node scheduling algorithms in sensor networks a very challenging problem.

In this paper we study randomized node scheduling algorithm for two kinds of sensor networks, namely battery-driven and rechargeable sensor networks. The randomized algorithms considered in this paper are completely distributed in nature and do not require the nodes to exchange any messages. For the battery-driven sensor networks we evaluate the coverage and connectivity properties achieved by the randomized scheduling algorithms. For the rechargeable sensor networks we examine the time-average utility achieved by the randomized scheduling algorithm.

A. Project Goals

**Concentration results for sensor coverage:** Although the coverage properties of randomized scheduling algorithm in battery powered sensor network has been already studied, we revisit the coverage problem. Through a novel formulation we provide concentration results that bound the deviation of fraction of area covered at any time from its expected value.

**Connectivity properties of randomized scheduling:** The connectivity properties of the disjoint subsets of sensors constructed by randomized scheduling is not well understood. Depending upon the application of WSN, it is important that either the disjoint sets are fully connected or a sensor is able to communicate with a central sink node using a path that consists of sensors belonging only to its sensor set. In some applications such as surveillance and motion tracking it is necessary that the sensors are able to share the sensed information among themselves or communicate the information to a central sink in a real time manner. Thus connectivity is a critical factor...
for the WSN to achieve its goals. In the paper we evaluate the probability that the disjoint subsets created by the randomized algorithm are fully connected. The technique used may be extended to find the probability of a sensor set being connected to a sink node. We also provide an alternative formulation for investigating the connectivity of sensor set with a central sink node.

**Scheduling of rechargeable sensors**: We propose a randomized scheduling algorithm for the rechargeable sensor networks that (unlike existing schemes) does not require nodes to exchange any messages. We provide analysis of the time average utility achieved by the scheduling algorithm. For the analysis we only consider the independent lifetime model, where lifetime of one sensor does not depend on that of other sensors that are active at the same time. We use simulations in order to investigate the performance of the correlated lifetime model, where lifetime of a sensor depends on that of other sensors that are active at that time.

**B. Paper Outline**

The rest of the paper is organized as follows. In Section II we present a brief overview of related work. In Section III we discuss the coverage and connectivity properties of the randomized scheduling algorithm in battery-driven sensor networks. The randomized scheduling algorithm and its performance for rechargeable sensor networks is presented in Section IV. We discuss our results and conclude in Section V.

**II. PREVIOUS RESULTS**

We first discuss the previous results for the coverage problem for battery driven sensor networks followed by previous results for rechargeable sensor networks.

**A. Previous Results for Battery Driven Sensors Networks**

The issue of coverage has been studied extensively in the literature. Area coverage, where the goal is to monitor a specified region, has been considered in [25], [29], [27]. Target (or point) coverage has been studied in [3], [5], [4], [12], [6]. Coverage has also been studied from the perspective of maximal support (or breach) path in [20], [24].

In [6], deterministic sensor placement allows for topology-aware placement and role assignment, where nodes can either sense or serve as relay nodes. Zhang and Hou [29] show that if the communication range of sensors is at least twice as large as their sensing range, then coverage implies connectivity. They also develop some optimality conditions for sensor placement and develop a distributed algorithm to approximate those conditions, given a random placement of sensors.

An important method for extending the network lifetime for the area coverage problem is to design a distributed and localized protocol that organizes the sensor nodes in sets. The network activity is organized in rounds, with sensors in the active set performing the area coverage, while all other sensors are put in the sleep mode. Set formation is done based on the problem requirements, such as energy-efficiency, area monitoring, connectivity, etc. Different techniques have been proposed in literature [27], [29] for determining the eligibility rule, that is, to select which sensors will be active in the next round. This notion of classifying the sensor nodes into disjoint sets, such that each set can independently ensure coverage and thus could be activated in succession, has been considered in [3], [25]. Cardei and Du [3] show that the disjoint set cover problem is NP-complete and propose an efficient heuristic for set cover computations using a mixed integer programming formulation. A similar centralized heuristic for area coverage has been proposed in [25], where the region is divided into multiple fields such that all points in one field are covered by the same set of sensors. Then, a most-constrained least-constraining coverage heuristic is developed which is empirically shown to perform well. In [5] the constraints for the set of sensors to be disjoint and for these sets to operate for equal time intervals, are relaxed and two heuristics, one using linear programming and the other using a greedy approach are proposed and verified using simulation results.

Randomized scheduling of sensor nodes in order to provide prolonged coverage in the sensor network has been considered in [2], [17], [18]. Two important motivations for considering a randomized scheduling algorithm are: (i) The disjoint set cover problem is NP-complete and hence it is not possible to solve it optimally using any centralized deterministic algorithm in polynomial time, and (ii) Randomized scheduling algorithms could be easily implemented in a distributed network environment, since they require the sensors to maintain minimal or no global state information, and also avoid unnecessary communications overhead for decision making amongst the sensor nodes.

The basic idea is to divide the sensor nodes into $K$ disjoint sets, such that each of the sets is independently able to provide sufficient coverage in the network. Then each of the sets is activated in succession in the network. Such an activation schedule is desirable in the presence of redundant deployment of sensor nodes, and it prolongs the network lifetime, which is actually proportional to $K$, the number of such sets of sensors. A randomized schedule puts a sensor $s$ into one of the cover sets, with probability that $s$ joins a particular cover set given by $\frac{1}{K}$.

A randomized scheduling of sensor nodes is analyzed in [17] and the expected values of coverage intensity achieved in the network through random cover set formations is provided. Given the total number of sensor nodes in the network, they also provide an upper bound on the total number of disjoint cover sets possible, to obtain a desired average (or expected) coverage intensity. In [18], the authors suggest turning on extra nodes in a cover set in order to provide connectivity in the network, and evaluate the performance of their schemes using simulations.

[2] allows cover sets to cover area elements partially and provides bounds on the number of times an area element is covered by the randomized algorithm in comparison with the maximum number of times it could be covered using a centralized deterministic algorithm. The sensors are partitioned such that the expected coverage is within a factor of $1 - \frac{c}{2}$ of the optimal. They also provide some guarantees on the number of times the least covered area element is covered.
The connectivity properties of the cover sets created by randomized algorithm has not been studied previously. Also sufficient concentration results on coverage achieved by the cover sets and a methodology to choose “good” $K$, the number of disjoint sets of sensors to use, have not been provided previously.

Our model and formulation is different from previous work. We provide concentration bounds on coverage achieved by the cover sets formed. In addition, we study the effect of choosing various values of $K$ on the coverage properties of the cover sets formed. For characterizing the connectivity property of a cover set, we evaluate the probability with which any pair of sensors belonging to a cover set are connected and provide a formulation to study the connectivity of a cover set with a central sink node. We also discuss the effect of the ratio of communication and sensing radius on the connectivity of the cover sets if $K$ is chosen so as to provide “good” coverage.

B. Previous Results for Rechargeable Sensors Networks

Node activation schedules to provide a better quality of coverage in rechargeable sensor networks have been considered in [8], [13], [14], [10]. In the case of rechargeable sensors, the network lifetime is no more the desired metric to optimize, in [8], [13], [14], [10]. In the case of rechargeable sensors, average in rechargeable sensor networks have been considered recharge of its battery, and show that a simple threshold based node activation policy achieves asymptotically optimal performance with respect to the sensor energy bucket size, under various correlation system models.

[13], [14] consider a restrictive sensor system model, wherein a sensor node could be activated only upon complete recharge of its battery, and show that a simple threshold based node activation policy achieves performance more than \( \frac{3}{4} \) times the maximum achievable performance. [10], [8] consider sensor systems where a sensor could be activated as long as it has sufficient energy (partially rechargeable sensors), and show that a threshold based node activation policy achieves asymptotically optimal performance with respect to the sensor energy bucket size, under various correlation system models.

The previous work only considers threshold based policies in order to activate the sensors when they are available. The threshold based schemes require the sensors to continuously exchange information with each other in order to find out the number of sensors in active state. So in threshold based schemes the sensors are required to broadcast their state information throughout the area that they cover. If coverage area is very large than the communication area then the communication cost of such a broadcast may be prohibitive and may consume large portion of capacity of the network.

As opposed to the threshold based policies, the randomized activation policy considered in this paper does not require the sensor nodes to exchange any information. Hence the randomized activation scheme neither consumes any extra energy of the sensor nor wastes any capacity of the network.

III. RANDOMIZED SCHEDULING FOR BATTERY-DRIVEN SENSOR NETWORKS

The objective of the randomized scheduling algorithm for the battery-driven sensor networks is to divide the sensors into $K$ disjoint sets. Each disjoint set is referred to as a cover set. The union of all the cover sets must contain all the deployed sensors while intersection of any two cover sets must be a null set. Each cover set is scheduled for activation in a round robin manner. The lifetime of the sensor network using such a scheduling scheme is roughly $K$-times the lifetime of a single sensor. The randomized algorithm for partitioning the sensors into cover sets is the following: Each sensor joins a cover set from among $K$ cover sets by independently and uniformly generating a random integer between 1 and $K$.

We first investigate the coverage properties of cover sets constructed by the randomized algorithm mentioned above, followed by connectivity properties of the cover sets.

A. Coverage Properties

1) System Model: The sensor field is a torus\(^1\) with area $m$, which is divided into $m$ squares of unit area. Each unit square is called an area element. $N$ sensors are distributed uniformly at random and independently over the sensor field. An area element is said to be covered by a sensor if any point belonging to the area element is within the sensing range of the sensor. Let $r$ denote the sensing radius of each sensor and $d_i$ denote the number of area elements covered by sensor $i$. For large $r$ and $m$, that is when the dimensions of the grid are small compared to the dimensions of sensing radius and sensor field (define $d = \pi r^2$),

\[
    d_i \approx \pi r^2 = d \quad \forall \ i
\]

This sensing model is motivated by the fact that in many application the phenomenon being sensed exhibits large spatial correlation. So if the grid is fine enough, then knowledge about the phenomenon in an area element may be inferred by looking at only a part of the area element. This model is illustrated in Figure 1. The correspondence between sensors and the area elements covered by them may be represented by a bipartite graph, as shown in Figure 2. Here, the out-degree of each node on the left hand side of the graph, representing the sensors, is at least $d$. We assume that $Nd/m$ is an integer and $Nd/m \gg 1$. Note that, in the average case an area element is covered by sufficiently large number of sensors ($\approx Nd/m$).

The coverage metric considered in this paper is the number (fraction) of area elements covered by a cover set. We will evaluate the expected number of area elements covered by a cover set and use Chernoff’s bound to provide concentration results.

2) Coverage Analysis: The coverage property of the randomized scheduling algorithm would definitely depend upon the number of cover sets used. Large number of cover sets would lead to higher network lifetime but at the same time lower fraction of area elements would be covered by each

\(^1\)We choose torus area in order to avoid the boundary conditions faced in a square sensor field. Asymptotically the results would hold true for the square area as well.
Area element covered by a sensor

Fig. 1. The coverage model.

Area elements covered by a sensor

Fig. 2. Representation of area elements covered by sensors as a bipartite graph.

The asymptotic result follows from the fact that $\lim_{N \to \infty} \left(1 - \frac{m}{N}\right)^N = \frac{1}{e}$. Thus if the number of sensors deployed is very large and $K$ is chosen to be equal to $\frac{Nd}{m}$, then the probability that an area element is not covered by a cover set is given by $\frac{1}{e}$. In the rest of this section we will discuss only the asymptotic case when $N \to \infty$.

**Corollary 1:** Let $O_k$ denote the number of area elements covered by the cover set $k$, then in the asymptotic case as $N \to \infty$,

$$E[O_k] = m - \frac{m}{e} \forall 1 \leq k \leq K$$  

Next, we derive a concentration result using Chernoff bounds for the number of area elements covered by a cover set.

**Lemma 2:** With probability at least $1 - \delta$ ($0 < \delta < 1$), the following inequality holds for $O_k, \forall k : 1 \leq k \leq K$.

$$m - \frac{m}{e} - \sqrt{\frac{m}{2} \log \frac{2}{\delta}} \leq O_k \leq \min \left( m - \frac{m}{e} + \sqrt{\frac{m}{2} \log \frac{2}{\delta}}, m \right)$$  

**Proof:** Since $O_k = \sum_{i=1}^{m} z_{ik}$, $E[O_k] = m(1 - \frac{1}{e})$. Using Chernoff bounds, we have,

$$P[|O_k - E[O_k]| \geq mt] \leq 2e^{-2mt^2}$$  

Put $\delta = 2e^{-2mt^2}$ to get, $mt = \sqrt{\frac{m}{2} \log \frac{2}{\delta}}$. Thus,

$$P \left[ O_k - m + \frac{m}{e} \geq \sqrt{\frac{m}{2} \log \frac{2}{\delta}} \right] \leq \delta$$  

With probability at least $1 - \delta$, we have,
\[|O_k - m + \frac{m}{e}| \leq \sqrt{\frac{m}{2} \log \frac{2}{\delta}} \] (13)

Or,

\[m - \frac{2m}{e} \leq O_k \leq m - \frac{m}{e} + \sqrt{\frac{m}{2} \log \frac{2}{\delta}} \] (14)

Also, \(O_k \leq m\), the total number of area elements. Hence the result.

Choosing \(\delta = 2e^{-2m}e^{-a}\), we get

\[m - \frac{2m}{e} \leq O_k \leq m \] (15)

with probability at least \(1 - 2e^{-2m}e^{-a}\), which tends to 1 for large \(m\). Thus, the number of area elements covered by a cover set formed using the randomized set selection is greater than \(m - \frac{2m}{e}\), w.h.p.

Next we comment on the effect of the number of cover sets, \(K\), on the coverage achieved by the randomized scheduling.

**Effect of \(K\) on \(O_k\):** Suppose that instead of choosing \(K = \frac{N_d}{m}\), we choose \(K = \alpha \frac{N_d}{m}\). Then, equation (2) becomes

\[P[z_{ik} = 1] = 1 - \left(1 - \frac{1}{N \alpha}\right)^N \] (16)

and equation (3) becomes

\[\lim_{N \to \infty} P[z_{ik} = 1] = 1 - \frac{1}{e^{1/\alpha}} \] (17)

Thus,

\[E[O_k] = m - \frac{2m}{e^{1/\alpha}} \] (18)

and the concentration result corresponding to equation (15) becomes

\[m - \frac{2m}{e^{1/\alpha}} \leq O_k \leq m \] (19)

with probability at least \(1 - 2e^{-2m}e^{-a}\).

We observe that by increasing the number of cover sets, \(K\), by a factor \(\alpha\), the probability that an area element is not covered by a cover set is increased by a factor of \(e^{1-\frac{1}{\alpha}}\), while the network lifetime, which is proportional to \(K\), is increased only by a factor \(\alpha\). Thus, the quality of coverage of a cover set deteriorates exponentially with \(\alpha\), while the network lifetime only increases linearly with \(\alpha\).

For large \(m\), better concentration results may be obtained by choosing \(\alpha < 1\). For example if \(m \gg e^4\) and we choose \(\alpha = 0.25\), then from equation (19), at least 96 percent of the area elements would be covered w.h.p. by each cover set.

**B. Connectivity Properties**

In this section we investigate the connectivity property of the cover sets returned by the randomized algorithm. Intuitively the bound on probability that a cover set is connected would depend on the communication radius of the sensors and the number of cover sets used. If the number of cover sets are chosen so as to satisfy the coverage quality (e.g. \(K = N_d/m\) as used in the last section), then the connectivity of a cover set would depend upon the ratio of communication and coverage radius. Let \(r_c\) denote the communication radius of a sensor and let \(\beta\) denote the ratio of coverage radius to communication radius. That is

\[r = \beta r_c\] (20)

Also it follows that

\[\pi r_c^2 = \pi r^2 = \frac{d}{\beta^2} \] (21)

or the coverage area of a sensor is \(\beta^2\) times its communication area.

Let \(N_i\) be the number of sensors in the cover set \(i\), \(1 \leq i \leq K\). We know that

\[\sum_{i=1}^{K} N_i = N \] (22)

and

\[E[N_i] = \frac{N}{K} \forall 1 \leq i \leq K \] (23)

Like the last subsection, we first use \(K = N_d/m\) in order to get asymptotic results and later discuss how scaling \(K\) with a factor \(\alpha\) affects the connectivity properties. For \(K = N_d/m\), \(E[N_i] = m/d\). In order to obtain asymptotic expected value of the probability that a cover set is connected we consider an “average” cover set with \(N_i = m/d\) sensors. Let \(A\) denote the \(N_1 \times N_1\) adjacency matrix of such a cover set.

From [22], we know that a path of length \(l\) exists between sensors \(i\) and \(j\) belonging to the cover set if \(A_{ij}^l = 1\) (\(A_{ij}\) is the \(i, j\) element of \(A^{l}\)), where the product operator is the boolean AND operator instead of normal multiplication. Thus sensors \(i\) and \(j\) belonging to a cover set are connected if

\[\sum_{l=1}^{N_1-1} A_{ij}^l = 1 \] (24)

where all the products and sums are boolean (AND and OR) operators. We sum up to \(l = N_1 - 1\) because if the size of cover set is \(N_1\), the length of path between any two nodes could be at most \(N_1 - 1\).

**Lemma 3:** Let \(p_{ij}^{(l)}\) denote the probability that nodes \(i\) and \(j\) belonging to a cover set are connected by a path of length \(l\), then in the asymptotic case

\[p_{ij}^{(1)} = \frac{d}{m \beta^2} \] (25)

and

\[p_{ij}^{(l)} = 1 - e^{-\frac{p_{ij}^{(l-1)}}{\beta^2}} \quad \forall 2 \leq l \leq N_1 - 1 \] (26)

for all \(1 \leq i \neq j \leq N_1\).

**Proof:** Note that

\[p_{ij}^{(l)} = P[A_{ij}^l = 1] \]

\(A_{ij} = 1\) if sensors \(i\) and \(j\) lie within communication range of each other. This happens if \(i\) lies within disk of radius \(r_c\) centered at \(j\). Since the sensors are distributed uniformly at random over the torus of area \(m\), probability that \(A_{ij} = 1\) equals \(\pi r_c^2/m\). Using \(\pi r_c^2 = d/\beta^2\) we get (25). Thus matrix \(A\) may
be viewed as the adjacency matrix of a random graph where probability that an edge exists between nodes $i$ and $j$ is given by $\frac{d}{\beta^2 m}$.

Now consider $p_{ij}^{(l)}$ for $l \geq 2$. $A_{ij}^l$ is given by

$$A_{ij}^l = \sum_{k=1}^{N_i} A_{ik}^{l-1} A_{kj}$$

Now probability that $A_{ik}^{l-1} A_{kj} = 1$ is given by

$$P[A_{ik}^{l-1} A_{kj} = 1] = p_{ik}^{(l-1)} \cdot P[A_{kj} = 1] = p_{ik}^{(l-1)} \frac{d}{\beta^2 m}$$

Since $i$, $j$, $k$ are arbitrary sensors, due to the symmetry of formulation, we have $p_{ik}^{(l-1)} = p_{ij}^{(l-1)}$. Thus probability that $P[A_{ik}^{l-1} A_{kj} = 0] \forall 1 \leq k \leq N_1$ equals $(1 - p_{ij}^{(l-1)} \frac{d}{\beta^2 m})$. This implies that

$$P[A_{ij}^l = 1] = 1 - \left(1 - p_{ij}^{(l-1)} \frac{d}{\beta^2 m}\right)^{N_i}$$

Substituting $N_i = \frac{m}{d}$ and taking the limit $\frac{m}{d} \to \infty$ we get the following asymptotic result

$$p_{ij}^{(l)} = \lim_{\frac{m}{d} \to \infty} 1 - \left(1 - p_{ij}^{(l-1)} \frac{d}{\beta^2 m}\right)^{\frac{m}{d}} = 1 - e^{-p_{ij}^{(l-1)} \frac{d}{\beta^2}}$$

where we use the standard limit $\lim_{x \to -\infty} (1 - \frac{x}{\beta})^x = e^{-\alpha}$. This proves (26).

The following corollary gives the probability that two sensors belonging to a cover set are connected.

**Corollary 2:** In the asymptotic case, the probability that the sensors $i$ and $j$ belonging to a cover set are connected, denoted by $p_{ij}$, is given by

$$p_{ij} = 1 - \left(1 - \frac{d}{\beta^2 m}\right)^{\frac{m}{d}}$$

**Proof:** Sensors $i$ and $j$ are connected if $\sum_{l=1}^{N_i-1} A_{ij}^l = 1$. Now $P[\sum_{l=1}^{N_i-1} A_{ij}^l = 0]$ is given by

$$P[\sum_{l=1}^{N_i-1} A_{ij}^l = 0] = \left(1 - \frac{d}{\beta^2 m}\right) \prod_{l=2}^{N_i-1} (1 - p_{ij}^{(l-1)})$$

$$= \left(1 - \frac{d}{\beta^2 m}\right) \prod_{l=2}^{N_i-1} \exp(-p_{ij}^{(l-1)} \frac{d}{\beta^2})$$

$$= \left(1 - \frac{d}{\beta^2 m}\right) \exp\left(-\sum_{l=2}^{N_i-1} \frac{d}{\beta^2} p_{ij}^{(l-1)}\right)$$

Now $p_{ij}$ is given by

$$p_{ij} = 1 - P[\sum_{l=1}^{N_i-1} A_{ij}^l = 0]$$

which leads to (28).

The probability that two nodes belonging to a cover are connected, $p_{ij}$, depends on (i) $\frac{d}{m}$ – which is inversely proportional to the expected number of sensors in a cover set; and (ii) $\beta^2$ – the square of the ratio of coverage radius to communication radius of sensors. It is observed that $p_{ij}$ increases as $\frac{d}{m}$ and $\beta^2$ decrease. As $\frac{d}{m}$ decreases, the expected number of sensors in a cover set also increases, thus probability that a cover set is partitioned into disconnected components decreases. The dependence of $p_{ij}$ on $\beta$ is much more complex. The number of cover sets $K$ is chosen such that the sensor field is covered with high probability. If a cover set covers the entire area with high probability then this implies that the union of disks corresponding to the coverage area of sensors belonging to a cover set equals the entire sensor field with high probability. Thus if communication radius of sensors is much larger than the coverage radius (i.e. small $\beta$) then the union of disks corresponding to the communication area of sensors belonging to a cover set would also be equal to sensor field with high probability. Thus for small $\beta$ the cover set is connected with high probability.

Figure 3 shows how $p_{ij}$ varies with $\beta$. It is observed that if $\beta < 1$, a cover set is connected with probability close to one. However $p_{ij}$ drops sharply around $\beta = 1$ and becomes almost 0 for $\beta > 3$. Therefore if the number of cover sets is chosen based on coverage criterion only, the resulting cover sets have good connectivity only if ratio of coverage radius to communication radius is small ($< 1$).

Up to now we have discussed results for the case where $K = N d/m$. We will now briefly discuss the effect of scaling the number of cover sets by a factor $\alpha$. If $K = \alpha N d/m$, then probability that two nodes belonging to a cover set are connected is given by the following Corollary.

**Corollary 3:** If number of cover sets, $K$, equals $\alpha N d/m$ then

$$p_{ij}^{(1)} = \frac{d}{\alpha \beta^2 m}$$

Fig. 3. Plot of $p_{ij}$ versus the $\beta = \frac{r}{r_c}$. It is observed that if $r_c > r$, then two nodes belonging to a cover are connected with high probability. The probability drops sharply when $r_c \leq r$. 
\[ p_{ij}^{(l)} = 1 - e^{-\frac{p_{ij}^{(l-1)}}{\alpha \beta^2}} \quad \forall \ 2 \leq l \leq N_1 - 1 \] (31)

and

\[ p_{ij} = 1 - \left( 1 - \frac{d}{\beta^2 m} \right) \exp \left( -\frac{\sum_{k=1}^{N_1-2} p_{ij}^k}{\alpha \beta^2} \right) \] (32)

for all sensors \( i \neq j \) belonging to the same cover set.

**Proof:** The change in the number of cover sets does not affect the probability that two sensors belonging to a cover set lie within each others communication range (which depends only on the communication range). Thus \( p_{ij}^{(l)} \) remains unchanged and hence (30) is same as (25).

Since \( K = \alpha N d / m \), \( E[N_1] = \frac{m}{\alpha d} \) and hence equation (27) is modified to

\[ p_{ij}^{(l)} = \lim_{n \to \infty} \frac{1}{n} \left( 1 - p_{ij}^{(l-1)} \right) \left( 1 - \frac{d}{\beta^2 m} \right)^{\frac{m}{\alpha d}} = 1 - e^{-\frac{p_{ij}^{(l-1)}}{\alpha \beta^2}} \] (33)

which leads to (31). Following procedure as used in the proof of Corollary 2 and plugging in (30) and (31), we get (32).

Corollary 3 implies that the exponential part of the probability of a pair of sensors belonging to a cover set not being connected is raised to the power of 1/\( \alpha \) as the number of cover set is increased by a factor \( \alpha \). Thus if \( K = 0.5 N d / m \), then

\[ p_{ij} = 1 - \left( 1 - \frac{d}{\beta^2 m} \right) \exp \left( -\frac{\sum_{k=1}^{N_1-2} p_{ij}^k}{\alpha \beta^2} \right) \] . Not surprisingly this result is similar to that observed for coverage achieved by a cover set. However the rate of decrease of \( p_{ij} \) with decreases in \( \alpha \) is smaller.

In the above section we investigated the asymptotic probability that any two sensors of cover set are connected to each other. In certain scenarios the connectivity of a cover set is not an important issue. Instead it is important that each sensor is connected to a sink station. The requirement of connectivity with a sink can be easily incorporated in the above analysis by including the sink node in every cover set. We refer to the set obtained by adding the sink node to a cover set as augmented cover set. Now notice that if each sensor is connected to the sink node then the augmented cover must be fully connected, since any two sensors are now guaranteed to have a path through the sink node if a direct path did not exist between them in the cover set. Thus the above analysis can be extended to the cases where only the connectivity of sensors to sink node is of interest by considering augmented cover set instead of cover sets. We now present an alternative formulation for investigating the connectivity property of a cover set when connectivity of sensors to the sink node is of interest. The alternative formulation is not easy to analyze and hence we present only an outline on how to proceed with the analysis.

**Alternative formulation for connectivity of sensors to a sink node:** Consider a scenario where there is a sink node which all the active sensors need to be connected with. A cover set is said to be connected if all the nodes belonging to the cover set are able to communicate with the sink node.

Consider a sensor, say sensor \( i \). A path from sensor \( i \) to the sink may be represented as a sequence of sensors such that the first sensor in the sequence is \( i \), the last sensor is the sink (s) and each sensor in the sequence lies within communication range of the preceding sensor. Let \( P(i) \) be the collection of all disjoint paths in the sensor network (when all sensors are active). Two paths are said to be disjoint if there exists at least one node that is not common to both paths. Let \( M_i \triangleq |P(i)| \) and let \( \{i, x_{i1}, x_{i2}, \ldots, x_{ij}, s\} \) denote the \( j \)th path \( (1 \leq j \leq M_i) \), where \( l_j \) is the number of intermediate nodes between sensor \( i \) and the sink node in the \( j \)th path.

Without loss of generality, we assume that sensor \( i \) belongs to cover set 1. We define a random variable \( Y(j) \) such that \( Y(j) = 1 \) if sensor \( j \) belongs to cover set 1 and 0 otherwise. Thus for \( j \neq i \),

\[ Y(j) = \begin{cases} 1, & \text{w.p.} \frac{1}{K} \\ 0, & \text{w.p.} 1 - \frac{1}{K} \end{cases} \] (34)

Let \( Z_i \) denote a random variable indicating whether \( i \) is connected to the sink or not, i.e. \( Z_i = 1 \) if there exists a path comprising of sensors in the cover set 1 that connects sensor \( i \) to the sink. \( Z_i \) can be written as

\[ Z_i = \prod_{j=1}^{M_i} Y(x_{kj}) \] (35)

where the product and summation in the above equation are boolean (AND and OR) operations respectively.

So finding \( P[Z_i = 1] \) boils down to finding the probability that the satisfiability problem represented by the r.h.s. of equation (35) is satisfied by random assignment of variables \( Y(j) \) according to equation (34). This would depend on \( M_i \), \( K \) and \( l_j \), which in turn would depend on the spatial distribution of the sensors and the communication radius of the sensors \( (r_c) \). It is simple to see that

\[ P\left[ \prod_{k=1}^{l_j} Y(x_{kj}) = 1 \right] = \left( \frac{1}{K} \right)^{l_j} \quad \forall \ 1 \leq j \leq M_i \] (36)

If all the paths are independent (i.e. no two paths share a common sensor), \( P[Z_i = 1] \) is given by

\[ P[Z_i = 1] = 1 - \prod_{j=1}^{M_i} \left( 1 - \left( \frac{1}{K} \right)^{l_j} \right) \] (37)

However since the values of each product on the r.h.s. of equation (35) is not independent, therefore calculation of \( P[Z_i = 1] \) is non-trivial. Also the dependence of \( M_i \), \( K \) and \( l_j \) on network parameters is not immediately obvious. We leave the exact characterization of connectivity of cover sets with sink node as a part of future work.

**IV. RANDOMIZED SCHEDULING FOR RECHARGEABLE SENSOR NETWORKS**

As shown in figure 4, a rechargeable sensor has three states:

1) **Active state:** If a sensor is in active state and an event occurs then the sensor detects the event with probability \( p_d \). When the total energy of the sensor is consumed then the sensor makes transition into recharge state.

2) **Recharge state:** In this state the sensor does not sense any event, but only replenishes its power supply. The
power supply is fully replenished when the sensor collects $K_{max}$ quanta of energy after which it moves to ready state.

3) **Ready state**: This state corresponds to the sleep state of the battery-powered sensors. In this state the sensor consumes negligible energy and waits to be activated.

As shown in [13] and [14], the coverage performance depends on how a sensor decides to migrate from ready state to active state. Let $N$ and $n(t)$ denote the number of sensors deployed and the number of active sensors at time $t$ respectively. For simplicity we assume that the entire sensing field is within sensing range of every sensor. The utility at time $t$, $U(n(t))$ is equal to $1 - (1 - p_d)^{n(t)}$. The utility function is simply the probability that an event occurring at time $t$ is sensed by at least one active sensor. The time average utility is given by

$$\bar{U} = \lim_{N \to \infty} \frac{1}{T} \int_0^T U(n(t)) dt$$

Note that if $U(n(t))$ is ergodic then $\bar{U}$ simply equals the $E[U(n(t))]$, the expected value of $U(n(t))$.

The policy used for deciding the migration of sensors from ready state to active state would decide $n(t)$. Threshold based activation policies are studied in [13] [14], where a sensor in ready state is activated at time $t$ if $n(t)$ is less than some threshold. It is shown that under certain conditions there exists an optimal threshold which maximizes $\bar{U}$. Such threshold based policy avoids simultaneous activation of all the sensors which might lead to alternating periods of large number of activated sensors and very few activated sensors. However the threshold based policies have few drawbacks. They require message exchanges in order to find out the number of active sensors. They also require the communication radius to be at least twice as large as the sensing radius, in order for the sensors to communicate with their neighbors. The randomized activation policy proposed in this paper does not have these drawbacks.

Before presenting the randomized policy and its analysis, we first present the lifetime models. Throughout this section we consider only discrete time event models, where time is indexed by $t = 0, 1, 2, \ldots, \infty$. For the discrete time case the time average utility is reduced to

$$\bar{U} = \lim_{L \to \infty} \frac{1}{L+1} \sum_{t=0}^L U(n(t))$$

**Lifetime Model**: Three lifetime models are considered in this paper:

1) **Independent Lifetime (IL) Model**: In IL model the discharge and recharge times of all the sensors are independent. While in active state, a sensor looses one quantum of energy with probability $q_a$ at each time step. Thus the expected number of time steps spent by a sensor in active state equals $K_{max}/q_a$. In recharge state a sensor gains one quantum of energy with probability $q_r$ at each time step and the expected time spent by a sensor in recharge state equals $K_{max}/q_r$. Also $q_r \ll q_a$, which ensures that recharge rate is much slower than the discharge rate.

2) **Deterministic Lifetime (DL) Model**: In DL model, each sensor remains in active and recharge state for $Q_a$ and $Q_r$ time steps respectively. This model reflects the scenarios where constant amount of energy is consumed during the active state (irrespective of event occurrence or communication) and the recharging source supplies uniform source of power during recharge state.

3) **Correlated Lifetime (CL) Model**: In CL model the discharge and recharge times of all the sensors are correlated. A typical correlated lifetime model is the following. With probability $q'_a$ all the sensors in active state consume one quantum of energy while with probability $q'_r$ all the sensors in recharge states gain one quantum of energy. This model reflects scenarios where one quantum of energy is consumed at all sensors and all sensors receive energy from the same source.

**Randomized Activation Policy**: In order to avoid communication requirement and restriction on communication radius, we propose the following randomized activation policy. At each time step each sensor in ready state makes a transition to active state with probability $p_t$ and remains in ready state with probability $1 - p_t$.

Figure 5 shows Markov chain representation of the randomized activation policy for various lifetime models. The hypothesis on which our work is based is that there exists an optimal value of $p_t$, denoted as $p_t^*$ (similar to the optimal activation threshold) that maximizes $\bar{U}$.

### A. Performance of the Randomized Activation Policy

1) **IL Model**:

**Lemma 4**: For the IL model the time average utility is given by

$$\bar{U}_{IL} = 1 - \left(1 - \frac{K_{max}/q_a}{K_{max}/q_a + K_{max}/q_r + 1/p_t}p_d\right)^N$$

**Proof**: Let $p_{Ai}(t)$ denote the probability that sensor $i$ is in active state at time $t$. Solving the Discrete time Markov chain shown in Figure 5(a), we get that under steady state

$$p_{Ai}(t) = \frac{K_{max}/q_a}{K_{max}/q_a + K_{max}/q_r + 1/p_t}$$

$$p_d$$
The expected value of \( U \) for all the active state at time \( 1 \) for all of utility, i.e.

\[
P[\mathcal{A}(t) = j] = \binom{N}{j} p_{\mathcal{A}(t)}(1 - p_{\mathcal{A}(t)})^{N-j}
\]

for all \( 0 \leq j \leq N \).

Thus utility at time \( t \), \( U(N_A(t)) \), is given by

\[
U(N_A(t)) = 1 - (1 - p_d)^{N_A(t)}
\]

The expected value of \( U(N_A(t)) \) is given by

\[
E[U(N_A(t))] = \sum_{j=0}^{N} 1 - (1 - p_d)^j \cdot P[N_A(t) = j]
\]

\[
= \sum_{j=0}^{N} 1 - (1 - p_d)^j \binom{N}{j} p_{\mathcal{A}(t)}^j (1 - p_{\mathcal{A}(t)})^{N-j}
\]

\[
= 1 - \sum_{j=0}^{N} \binom{N}{j} (p_{\mathcal{A}(t)}(1 - p_d))^j (1 - p_{\mathcal{A}(t)})^{N-j}
\]

\[
= 1 - (1 - p_{\mathcal{A}(t)}p_d)^N
\]

\[
= 1 - \left(1 - \frac{K_{\text{max}}/q_0 + K_{\text{max}}/q_r + 1/p_t}{p_d}\right)^N
\]

Since the Markov chain representing the states of the sensors is ergodic, the time average utility equals the expected value of utility, i.e.

\[
U_{IL} = \lim_{L \to \infty} \frac{1}{L+1} \sum_{t=0}^{L} U_{IL}(N(t)) = E[U(N_A(t))]
\]

which leads to (40).

2) DL Model:

Lemma 5: For the DL model, the time average utility is given by

\[
U_{DL} = 1 - \left(1 - \frac{Q_a}{Q_a + Q_r + 1/p_t}p_d\right)^N
\]

The proof of the above Lemma is similar to that of Lemma 4.

3) CL Model: For the CL model we were not able to generate any closed form expressions for the time average utility function. So we performed simulations in order to evaluate the dependence of \( U_{CL} \) on \( p_t \). Figure 6 shows how \( U_{CL} \) varies with \( p_t \).

4) Discussions: The results of Lemma 4 and 5 are obtained by steady state analysis of Markov chains shown in figure 5 and by using the fact that \( U(n(t)) \) is an ergodic process.

It should be noted that \( U_{IL}, U_{DL} \) and \( U_{CL} \) are all strictly increasing functions of \( p_t \). This implies that contrary to our hypothesis, there exists no optimal \( p_t \) and it is advantageous to always go from recharge state to the active state, with probability \( 1 \). This corresponds to the threshold activation policy with threshold equal to \( N \). However this activation policy is not optimal as shown in [13] and [14]. Hence avoiding communication overheads leads to a loss of performance in a rechargeable sensor network.

V. CONCLUSIONS AND FUTURE WORK

In this paper we investigated the performance of the randomized scheduling algorithms in wireless sensor networks. In particular we considered the coverage and connectivity properties of cover sets formed using randomized algorithms for partitioning sensors into cover sets and randomized activation algorithms for rechargeable sensor networks. We derived asymptotic values of the probability with which a cover set covers an area element and expected number of area elements covered by a cover set. Using Chernoff bound, we also present concentration results for the number of area elements covered by each cover set. We discuss the effect of scaling the number of cover sets on the coverage property and effective lifetime.
of the sensor network. It is found that if the number of cover sets is scaled by a factor $\alpha$ then the probability that an area element is not covered by a cover set is scaled by $e^{1-1/\alpha}$.

Next we investigated the connectivity property of a cover set. In order to characterize the connectivity of a cover set we use the adjacency matrix of the cover set ($A$) and use the fact that if the cover set is fully connected then all the off-diagonal elements of $A = \sum_{i=1}^{N} A^l$ equal 1 when all the summations and products are binary AND and OR operators respectively. Using this we obtain asymptotic value of $P[A_{ij} = 1]$ i.e. probability that any two sensors of a cover set are connected. We found that if $\beta$ is ratio between coverage radius and communication radius and the cover sets are chosen so as to satisfy coverage with high probability then $P[A_{ij} = 0]$ varies as $e^{-1/\beta^2}$ i.e. if communication radius is much larger than the coverage radius, the cover sets are connected with high probability. The effect of scaling the number of cover sets on the connectivity of a cover set is similar to the effect on coverage of a cover set.

We started investigating the randomized activation policy for rechargeable sensor networks based on the hypothesis that there exists an optimal randomized activation policy that maximizes the life-long utility of the rechargeable sensor network. We evaluated its performance using discrete time Markov chain analysis and simulations. It is observed that the optimal randomized activation policy is a trivial one, which activates the sensors as soon as they are recharged. There is still a lot of scope for future work related to scheduling algorithms for wireless sensor networks. All the results presented in this paper assume an initial uniform distribution of sensor nodes in the sensor field. When the initial distribution is uniform and the number of cover sets is carefully chosen, then randomized scheduling algorithm achieves very good coverage and connectivity. However a real deployment of a sensor network may not be uniformly distributed and might consist of regions of high and low sensor density. For such a scenario the randomized algorithm may lead to coverage holes and disconnected components. In such cases it is important that a sensor uses some local information, such as local density or location information, in order to decide which cover set to join. Cover set membership policies that allow communication and cooperation among one hop neighbors is also of much interest. For cases of uneven sensor distributions, the cover set membership policies should also allow a sensor to join multiple cover sets in order to avoid coverage holes or disconnected components. This is in contrast to the randomized schemes that allow a sensor to join only a single cover set.

REFERENCES